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Is Frege's Impasse concerning Predicative Functions Shared by Aristotle and Wittgenstein?

Abstract: *The idea of rejecting a certain form of transcendental realism fits perfectly with Wittgenstein's and Aristotle's anti-realistic account of the world, and may serve in a humanly accessible form of realism to implement the resolution of Frege's epistemological impasses. Frege has taught us that it is possible to discover reality solely through the use of reason. Aristotle and Wittgenstein, on the other hand, have warned us that any such attempt at discovery is fraught with difficulty; it is the task of thinkers to painfully (even frustratingly) seek to uncover the order and laws underlying what really exists in the world. Frege's rationality is finally seen to be decreasing, as it takes refuge in meanings of concepts and results in his almost total silence as regards their predicative functions. Able, if reluctant, to understand the way out of the impasse in that there can be no other criteria than the ordinary ones for what is habitually done, the way suggested by Aristotle's use of the "square of oppositions", Frege paves the way for a philosophical understanding of Aristotle's and Wittgenstein's rejection of a robust form of realism in favour of a less demanding form of it.*

Keywords: *Realism/Non-realism argument, Square of oppositions, Wittgenstein, Aristotle's Anti-realism.*

The idea of rejecting a certain form of transcendental realism fits perfectly with Wittgenstein's and Aristotle's anti-realistic account of the world, and may serve in a humanly accessible form of realism to implement the resolution of Frege's epistemological impasses. But does this option recall Aristotle's "craftmanship" model of the world as we strive to turn the ideal into reality (without undercutting, however, the standard realist/non-realist argument) in search of a shift from reason's limitless explanatory power to more pragmatic attempts? How we understand terms and semantics for the said problems is stressed by Aristotle. Frege's ideas and assumptions as regards categorical inferences and propositions, as used in his "square of oppositions", have not overlooked the if and *must* key terms contained in it. This square, in fact, tells us whether or not things exist in the world. Frege, however, has taught us that it is possible to discover reality solely through the use of reason. Aristotle and Wittgenstein, on the other hand, have warned us that any such attempt at discovery is fraught with difficulty; it is the task of thinkers to painfully (even frustratingly) seek to uncover the order and laws

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I. Tools and Trends

Any advance made by Aristotle as regards any general thesis – such as that concerning truth and science – is always open to an obstinate "Why". Certainly we no longer expect answers in a literal sense for any superhuman intervention in the order of history to dismiss the "Why" question. What can be very dimly understood by this question is that we can be committed to realising the possibilities it may imply. This means that he could not proceed in terms of something which does not itself require to be explained. We cannot push this line of thought further here without mentioning that Aristotle slips into material reductionism, which entails a fragile understanding of the mental absorbed into the physical.

Again, Aristotle's inability to answer the question "Why does nature conform to given fixed ratios?" demonstrates that we cannot gain epistemic access into the real essences of things. This explains in his writings the material reductionist relation of the mental and the physical and the weakness in grasping the essences of biological kinds, as we can grasp terms for kinds without knowing how we come to grasp them. That we must necessarily lack a recognisable concept of the ideal, the world as it is in itself, is further stressed in his philosophy[1].

These being Aristotle's main lines of thought[2], there are a number of pressing issues in Frege: If there is an equinumeric collection (for example of knives and forks), the numbers (in this case, of knives and forks) must be identical. Does this assumption of modern logic overlook the *if* and *must* key terms which are contained in it? And does it tell us *whether or not such a collection exists in the world*? Frege has warned us that describing reality through the use of reason is very unlikely to succeed.

1 Charles 1997, 231-257.
2 Charles 2000, 206, n. 17.

One of the shortcomings of Frege's theory as regards concepts as a sub-class of functions, is that truth-values of predicative functional expressions do not work out for arguments (numbers) that substitute variables of these functions in the way he wishes them to. This leads to an impasse. Indeed, functions whose values are always a truth-value are highly problematic. As Russell has shown there is one concept that does not have a determinate value-range[3]. We shall examine this so-called "Russell's paradox" below. Hence, Frege's fundamental axiom that facts are simply true thoughts is invalid[4]. Frege thought that truth, being simple, cannot be reduced to anything simpler[5]. Does Frege borrow from Aristotle's use of the logical square here? What is the contribution of Wittgenstein as regards the idea of the square of oppositions? And how does Frege pave the way out of the impasse of predicate logic?

Certainly, following Frege, a would-be referent is a bound variable that results out of predication, i.e., out of abstraction[6]. One wonders, however, how the terms "true" and "false" can be made to apply to abstract concepts. Wittgenstein and Aristotle have arrived at the same impasse; however, they never consider the possibility of explaining, for example, y by the cause of y in the way that an idealist reductionism does; simply because if we claim to explain everything, all such explanations are similarly worthless. If we allow that y is caused by x and identify x as what causes y, then we take it that y is caused by the cause of y. Hence, If we accept that a would-be referent[7] is caused by the corresponding term of reference and identify this term as what causes the possible referent, then we achieve no more than tautology: a possible referent is caused by its cause. We can understand here that the criteria of identity regarding these terms are loose. Predicate and object coincide consistently only in one's mindset, according to Frege. And it is as if we postulate what Descartes would call an invisible, soundless ghost with no manifestations beyond the puzzling phenomenon of y, leading to worthless tautological explanations and an acceptation that y accounts for features of the world and our experience. This has no connection either with Wittgenstein or with Aristotle, who both insist that we need (even frustratingly) to explain reality in terms of something that itself needs no explanation. Is this a possibility for Frege also? Once examined through his use of the logical square, the possibility becomes a value which needs to be realised.

II. A Critique of Aristotle's and Wittgenstein's anti-realistic account of the world

In Aristotle's and Wittgenstein's logic, we start from the most abstract concept and proceed to the most concrete one. Hence, they both might have approved

3 Cobb-Stevens 1990, 69.
4 Cobb-Stevens 1990, 15, n. 36.
5 Cobb-Stevens 1990, 14, n. 33.
6 Linnebo 2018, 24.
7 Linnebo 2018, xii.

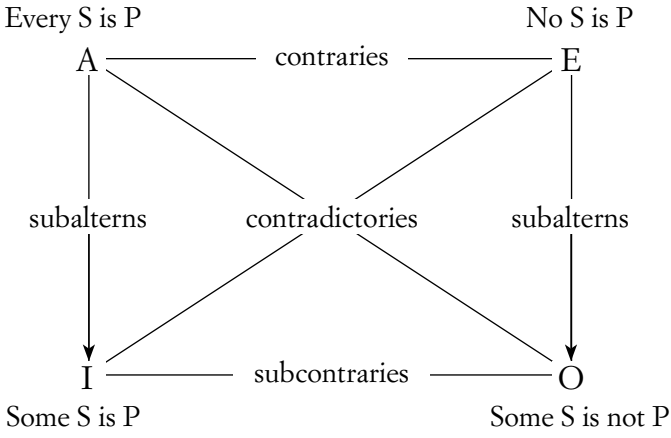
of Whitehead's idea that philosophy functions as the critic of abstractions. We must refer firstly to Aristotle's famous dictum[8]: what is, is, and what is not, is not, which leads us to the following question: Are universally quantified first principles commensurately universal in Aristotle? The negative reply makes us be at a loss to explain his dictum and to know how to implement his ideas.

This perspective being at work, as regards Aristotle's anti-realistic account of the world and its history, stresses his understanding to the mind-matter relationship. Aristotle succumbs to the view that talk about minds is nothing but talk about matter, which inevitably entails serious shortcomings[9]; especially solipsism and relativism[10]. But he does warn us to be on guard against them and thus make the best out of his anti-realistic account of the world.

This account concerns mainly that our idea of nature cannot go too far beyond the essentially human perspective of the experimentalist. Rationalistic values become non-sensical once we attempt to pursue them in abstraction from the concrete situation under which we can meaningfully realise them. No such a unitarian account is ever compiled by Aristotle[11]. He defines the categorical propositions in his *square logic* of oppositions in *De Interpretatione*[12], as conversion, obversion and contraposition. A proposition converts simply if it is necessarily equivalent in truth value to the proposition obtained by interchanging its terms. "Obverse" is the term used of a proposition immediately inferred from the original one, valid for all types of (A, E, I, O) categorical propositions. In a universal affirmative and in a universal negative proposition, the subject and the predicate term are both replaced by their negated counterparts. In contraposition one proposition is immediately inferred from another: having as its subject the contradiction of the predicate of the original logical proposition. Equivocation in the inference processes of conversion and obversion varies with different proposition types. Contraposition is distinct from conversion and obversion as regards its application. Frege was an inferentialist, at least in the sense that whatever content matters for logic is what is preserved through inference. We firmly hold, however, that Frege believed concepts to be self-contained, not in the sense that ideas are innate. However, this belief should be seen in the context of a rationalistic zeal, which proves in the end to have need of the restrictions and import referred to, among others, by Russell and Ayer. A thought certainly is not generated by thinking, Frege firmly holds[13]. But this only means that a thought exists prior to thinking; thus, he states, a fact is simply a true thought as understood with the *a priori* principles of a mathematical account of the world. Hence, he defines a number as an extension of concepts[14].

8 Aristotle 1984, *Metaphisics* b26-27.
9 Giouli 2023, 151-167.
10 Giuoli 2022c, 42-44, n. 19; 50-52, 71-72 and n. 70.
11 Charles 1997, 271-328.
12 Aristotle 1984, *De interpretatione* 2-3 and 6-7.
13 Frege 1969a, 137-163, esp. 149.
14 Frege 1956, § 106. Tr. §§106-109, 115-119.

The doctrine of *square logic* occurs in Aristotle. It begins in *De Interpretatione*[15], which contains three claims: that A and O are contradictories (one is the negation of the other), that E and I are contradictories (one is the negation of the other), and that A and E are contraries[16]: He calls an affirmation and a negation contradictory opposites when what one signifies universally the other signifies not universally. For example, every man is white – not every man is white, no man is white – some men are white. But he calls the universal affirmation and the universal negation contrary opposites, e.g., every man is just – no man is just. So, these cannot be true together, but their opposites may both be true with respect to the same thing, e.g., not every man is white – some men are white. This gives us the following fragment of the square for the categorical propositions from Aristotle's syllogistics (Stanford.library. sydney.edu.au):



A: Every *S* is a *P* $\forall x(Sx \rightarrow Px)$, E: No *S* are *P* $\forall x(Sx \rightarrow \neg Px)$, I: Some *S* are *P* $\exists x(Sx \,\&\, Px)$, Some *S* is not *P* $\exists x(Sx \,\&\, \neg Px)$. A and I are affirmative whereas E and O are negative. Two propositions are contraries if they cannot both be true. Two are subcontraries if they cannot both be false.

In *Prior Analytics*[17], Aristotle avers that in universal attribution it is necessary for the terms of the negative proposition (E) to convert; also, he adds, the particular affirmative (I) must convert in part. The doctrine of conversion per accidens follows[18] as regards A implying (but not implied by) I and E implying (but not implied by) O. The particular negative need not convert. The O form is the most

15 Aristotle 1984a, *De interpretatione*, 6-7.
16 Aristotle 1984a, *De Interpretatione*, 17 b 17-26.
17 Aristotle 1984a, *De Interpretatione*, 25 a5-10.
18 Aristotle 1984a, *De Interpretatione*, 25 a10-12.

problematic as it bears the weight of Aristotle's discussion of "infinite" negation[19] which is the use of negation to form a term from a term instead of a proposition from a proposition. Ackrill's translation[20] of O suggests rather 'Not every S is P' than "Some S is not P". With this wording, Parsons claims, Aristotle's doctrine automatically escapes modern criticism. If we assume that "S" is an empty term, this makes the I form "Some S is P" false. Its contradictory, the E form: "No S is P", is thus true, and this entails the O form in Aristotle's formulation: "Not every S is P", which must therefore be true. When the O form is worded "Some S is not P" this bothers us, but with it worded "Not every S is P" it seems plainly right, Parsons continues. "Every S is P" has existential import, and so if "S" is empty the A form must be false. But then "Not every S is P" should be true, as Aristotle's square requires. According to this view, he concludes, affirmatives have existential import, and negatives do not – a point that became elevated to a general principle in late medieval times. The ancients thus did not see the incoherence of the square as formulated by Aristotle because there was no incoherence to see.

Ayer[21], however, firmly believes that in Aristotelian logic propositions of the form A and E are treated as genuine contraries. They could both be false. But they could not both be true. One at least, however, of the A, E, I, O propositions is assumed to be true, he continues. This means that the existence of an S is assured *a priori*, no matter what it might be. Hence, he concludes, Aristotle's assumptions hold good under the false pre-supposition that some S's exist.

The point is not that we should endeavor to discredit Frege's model, proving it logically false, and then produce empirical evidence against it. That the logically false is not necessarily so, constitutes a motto of Fregean texts. This is a question of interest for George Vizyenos (March 8, 1849 – April 15, 1896), a member of the Greek man intelligentsia[22], also. No question arises about any fulfillment of, or negation of, the conditions for assertion. Not wishing to undercut the realism/non-realism argument, Frege opts to dedicate himself to realising the ideal of truth; not to claiming reality from the start. This implies a difference more fundamental than that between facts and the truth values of the propositions expressing them. This shows that the research question as phrased does not truly address the controversy suggested by the realist/non-realist argument in modern logic and the philosophy of language.

Frege's suggestions as regards the logical square, *if* true, provide the guarantee for this realisation of the ideal. We necessarily lack a recognizable concept of it. The ideal remains self-contradictory and over abstract. These suggestions, however, form his research question. Frege certainly does not claim that the ideal can be grasped intuitively in the sense of Descartes' clear and distinct apprehension. And Ayer's account of the false presupposition that some S's exist was undeniably his. Frege has realised that it does not follow that S is in fact true. But, although

19 Parsons 2021, n. 9.
20 Ackrill 1972-1973, 119-133.
21 Ayer 1973, 188.
22 Giouli 2022a, 36, 46, 122, 138-139, 145.

the premise is unprovable, we may answer that we cannot help believing it to be true[23]. However, the way Frege attempts to avoid monist solutions as regards the nature of the ideal, either as the mental or the physical, suggests[24] that he is moving in the direction of an exploration from the abstract to the concrete, but he definitely failed in his endeavour to establish this. There is nothing wrong in principle with exposing the difficulties of the uncompromising mind-body dualism and attempting to overcome them. To what extent however, Frege realises the importance of the if and must key terms as regard their use by Wittgenstein, remains unclear, despite the fact that Frege's *Begriffsschrift* also presents a square of oppositions, organised in an almost identical manner to the classical square, showing the contradictories, subalternates and contraries between four formulae constructed from universal quantification, negation and implication. Needless to add, Frege's square is parallel with Aristotle's square for syllogistics as regards our inability to have epistemic access to the real essences of things. Frege maintains[25] that a concept and its content, can be decomposed into arguments and functions. We should not match our judgements that concern the subject with our concepts that we form as the predicate of this subject. Instead, we need to decompose the judgeable content to arrive at the concept. But how to access the essence of reality is never mentioned by him.[26] Has he understood the importance of the if key terms of Aristotle's logic?

That existence is a property of concepts is admitted by Russell[27]. However[28], while the class of things which can be counted is itself something that can be counted, the class of men is not itself a man. The simplest way of presenting this antinomy (Russell's paradox) is by presenting the principle that the meaning of a propositional function is not specified until we specify the range of objects which can complete this function. That was the answer Russell gave to Frege; Frege replied that thus the foundations of mathematics had been undermined. This is easily understood because possible objects completing this function do not include anything which is defined in terms of the function itself. But Frege based his extensional account of reference and his theory of number on the axiom that for every concept there is a determinate value-range[29].

Here we see Aristotle's scorn of material reductionism, once we comment on "the class of things which can be counted being itself something that can be counted, whereas the class of men is not itself a man". Each individual, Aristotle states[30] among the many has a share of excellence and practical wisdom. When they meet together, they become in a manner one man. This "man" has many

23 Burke 1983, 41-42.
24 Burke 1983, 152.
25 Frege 1969b, 18.
26 Dummett 1981b, 333 and nn. 1-3.
27 Ayer 1972, 54.
28 Ayer 1972, 44-45.
29 Cobb-Stevens 1990, 69.
30 Aristotle 1984b, 1281 a40-1281 b10; 15-25.

feet and hands and senses. This holds with regard to their character and thought. Hence the many are better judges of music and poetry of than a single man. Some understand one part, he claims, and some another. And among them they understand the whole. We see here Aristotle's scepticism: it is doubtful whether the principles of collectivity and lack of ability can apply to every democracy, and to all bodies of men. In some cases, the principle cannot be held applicable, he states. The argument would equally hold about animals, and it will be asked if men differ from animals. There may be bodies of men, he admits, about whom our statement is true. If so, one difficulty has already been raised. There is also another difficulty: what power should be assigned to the mass of freemen and citizens who are not rich and have no personal merit? These comments certainly concern politics and philosophy. They do not tackle the ideal character assigned to mathematics by Frege. It is of abiding interest following Aristotle to add point to the lack of any recognizable concept of this ideal. Why should we not suppose that individual expertise will be submerged in general incompetence? Aristotle states that the multitude ought to be in power rather than the few best. This is true but it is only with some difficulty that this can happen. For the many, of whom not everyone is a good man, when they meet together may be better than the few good. The many are regarded not individually but collectively. A feast to which many contribute is better than a dinner provided out of a single purse.

Back to Russell again, we understand that we cannot say that two objects coincide when falling within the same extensional range of concepts. A concept for Frege is a function whose bound variables range over an unlimited spectrum. Hence, a concept determines truth-values for every conceivable object. Frege thought that the domain over which the variables of quantification range is the totality of objects, in the sense that every object either falls under the concept or not.

In brief, Russell's solution to the errors in Frege's logic[31] as regards the nature of concepts is that it is a different sort of entity (an object) that fills the argument-place attached to a concept. Hence, a concept is not self-determined. And objects (value-ranges), being complete entities, fall into the class of concepts. This empirical solution disappointed Frege. It only had the appearance of a solution: to say of the class of classes which are not members of themselves that it either is, or is not, a member of itself is neither true nor false but meaningless. So, Russell arrives at a system in which propositional functions[32] are arranged in a hierarchy. Objects that complete functions at a given level, constitute a *type*. Thus, what can be said (truly or falsely) about objects of one type, cannot meaningfully be said about objects of a different type. Thus, the statement of Epimenides the Cretan that propositions asserted by Cretans were all false is meaningless; unless he meant that all propositions of the order n which were asserted by Cretans were false. Since this would be a proposition of a higher order than n, no paradox arises.

31 Cobb-Stevens 1990, 70, n. 68.
32 Ayer 1972, 45.

All such mathematically inferred entities are, according to Russell, a logically elaborated set of constructions as abstracted out of the data present in experience. And our propositions about physical objects are analysable into collections of propositions about these data[33]. This is the way an abstraction must function in mathematics (not inversely, in Frege's romantic way).

Wittgenstein[34] attacks Frege's concept of the logical form of number attributions)[35], stating that not even a certain generality is essential to numerical statements. He offers the following example to clarify his critique: a correct description of the visual space in which 3 red circles are located on a blue background does not imply the occurrence of the expression "(∃x, y, z): x ε circular and red, y ε circular and red, etc". Wittgenstein further expands the example to prove that these circles are not true knowable objects; the red circles are what we observe, being patches of colour in visual space, tones etc. And although a thought is not necessarily a content of consciousness for either Frege or Wittgenstein[36], we see that reality does not exist unconditionally. The mind-independent world is altered by sense-perception. But even if reality does not exist unconditionally, this does not mean that it is inexistent. The medium of mathematics, however, may regulate order and reality, but it does not tell us whether or not there are any objects in the world[37]. This is exactly what Aristotle avers. Hence, we cannot perceive the ideal nor determine what it is to be real; undoubtedly, however, we can say that something is real[38]. For Whitehead, reality and primal facts can be (even contradictorily) whatever science requires them to be[39]. This is the bottom line for Aristotle also, despite his objectivity. Besides, it is the whole as a complete being, in his own words, that must be distinguished from the infinite: the divisible both in the direction of reduction and of addition[40]. We must mention at this point that no "supervenience" of the mental over the physical exists in Aristotle. Caston discusses the concept of supervenience physicalism[41]. True, the infinite (the non-finite) exists potentially in Aristotle in the way that matter exists in the sense in which we say (indefinitely, i.e., in a non-determined manner) "it is day" or "these are the games"; not independently in the way that what is finite (a whole man or a box) exists[42]. He adds, however, that "it is day" or "it is the games" are non-substantial beings: i.e., they are always different, yet *finite*[43]. To further clarify this and insist that infinite and finite do not coincide, he offers the example of matter being a part

33 Cottingham 1984, 103-104.
34 Wittgenstein 1979, MS 107, 11-12.
35 Hintikka and Hintikka 1989, 169-170.
36 Dummett 1981, 487.
37 Giouli 2020a, 122.
38 Burke 2000, 15-16. Whitehead 1926, 32-33.
39 Whitehead 1926, 32-33.
40 Aristotle 1984a, 207 a22-23; 206 a14-15.
41 Caston 1993, 122-126.
42 Aristotle 1984a, 206 b13-15.
43 Aristotle 1984a, 206 a30-35.

of the whole as bronze is a component of bronze statues[44]. At the same time, however, he tells us that *the infinite is limited by the great and the small that ought to contain intelligible things*[45] however absurd and impossible it is to suppose that unknowable and indeterminate infinite matter should contain and determine[46]. Thus, his claim regarding the limits and completeness of the infinite in virtue of something else refers solely to the size as limited by the great and the small; hence the mental "coincides" with the physical. The mental is the physical in Aristotle[47], as matter and form are one and the same. Nevertheless, in *Posterior Analytics*[48] one must recognise that authority lies with the world out there, not with mind's projections on that world. Hence, our attempts to divide reality are flawed: we cannot divide "what cannot be otherwise", according to Aristotle. Happening[49] and becoming constitute the endless flux of the history of ideas: as in an ever-running river flowing by and washing away any possibility of a distinctive view of it. Hence, Russell's idea of the dynamic procedures from the abstract to the concrete seems to the present writer to be valid[50]: objects are abstractions from the events in which they are realised[51]. One wonders, however, how Frege's truth values can be applied to abstract concepts at all. If we refer to the modern logical axiomatic, following Linnebo, we must defend predicative *vis-à-vis* impredicative abstraction, making a very strong case for admitting predicative abstraction over larger and larger domains, thus extending the availability of objects for quantification.

III. Existential requirements and logical square syllogistics

Reality[52] is not itself colourful or colourless, tasty or tasteless, sounding or soundless: we can perceive it only inductively[53]. We reach it by abstracting the above qualities from our perception. Behind this pattern of perception[54] there is no substantial ultimate cause. As nature fails to conform to given fixed ratios, it is not being governed by given causal principles. Essence and causation are thus inter-connected in Aristotle[55]. The reality of natural kinds, thus, is anchored in our craft engagement with instances of these kinds.

44 Aristotle 1984a, 207 a25-27.
45 Aristotle 1984a, 207 a29.
46 Aristotle 1984a, 207 a30.
47 Giouli 2020b, 87-101. Giouli 2020c, 183-196. Giouli 2021, 43-59.
48 Aristotle 1984a, 100 a15-b5.
49 Burke 2000, 28.
50 Giouli 2012a, 61-64, 74, 78.
51 Burke 2000, 29.
52 Burke 2000, 24.
53 Burke 2000, 28.
54 Burke 2000, 39.
55 Charles 2000, 260-265. Charles 1984, 234-242 and 213-227.

In terms of Fregean principles of abstraction, however, it is possible to argue that non- abstract pluralities of abstract objects are determined by equivalent relationships. These relationships suffice for these abstract objects to exist. Mathematical objects are *thin* in the sense that "very little is required for their existence". This conception of mathematical and other objects as "ultra-thin"[56] is criticised by Dummett: how can one make a reference to a concept, Dummett enquires. No genuine reference can be ensured[57]. We cannot demonstrate, he adds, that names given to abstract concepts have reference in the same way as names of concrete objects. We can only explain their use with the conception of them as "ultra-thin" without ascribing reference to them, Dummett concludes. Linnebo responds to this challenge, stressing that "semantically idle terms", to use Dummett's phrase, are not involved in meanings assigned to each sentence as a whole; but only through meaning assigned to its subsentential expressions. Thus, there is no direct interaction between the syntactic structure of a sentence and its semantic interpretation. Linnebo has also to respond to the question of whether his account of thin objects suggests a match of meanings and predication. He sets criteria of identity to endow singular terms with a reference. This, however, forms a compositional semantic theory. Compositionality[58], is a serious commitment for Frege.

Concept-defining no longer implies the extension of a concept, but its meaning. Does this affect Frege's use of the logical square? We cannot take refuge in the idea that since knives and forks on a table can be one-to-one correlated, this suffices for the existence of a number that represents the cardinality of both knives and forks[59]. Equinumerosity, however, automatically implies "unsaturated" predicative function.

We may claim in modern logic that numbers can exist[60] outside the content of a sentence. For Frege this is not taken to mean that a numberword signifies something when removed from the content of a sentence. Linnebo does not approve of this determining condition; it adds very little either to the ("easy") existence of being or to its demonstration[61]. The same is true regarding the appropriate use of criteria of identity. All claims that are necessary for reference according to Dummett are dismissed by a modern logician[62]: it is on the basis[63] of certain fundamental forms of reference on which all reference is ultimately based that criteria of identity are essential. Thus, modern internalism corresponds to Frege's internalism[64]. Dummett states[65] that Frege, believing in semantics, has constructed a general framework for this. Semantics in language must be internal to that language describing it from within, and not from some external viewpoint.

56 Linnebo 2018, 91-92 and n. 8.
57 Dummett 1981a, 499-500.
58 Dummett 1981a, 124.
59 Dummett 1981a, 4, n. 5.
60 Linnebo 2018, 111.
61 Linnebo 2018, xii.
62 Linnebo 2018, 33, n. 23.
63 Dummett 1981, 231-239.
64 Linnebo 2018, 42, n. 39.
65 Dummett 1955, 17.

This brings us to consider claims that to be an object means to be a possible referent of a singular term i.e., that to be a concept is a concept, which is absurd unless a criterion of identity for the would-be referent is provided. Frege, however, does not allow for identity standards. It is with the context principle that he attempts to explain cognitive access to numbers and other abstraction[66]. What Dummett avers is that impredicativity was "the serpent in Frege's paradise". Hence, the transformation of the generality of an equality into an equality is denied, despite Frege's claims that it is a possibility[67].

From 1890 onwards, Frege no longer regarded sentential contexts as being of importance[68]. Dummett, however, believes that if sentences are complete in structure, they retain the importance of the sentential contexts. Frege's idea is that sentences are a special kind of proper name, and thus are complete in their structure. Linking proper names and designating expressions with objects does not erase the problems: for example, the concept of justice will have to be regarded not as a name of a concept, but as a name of an object. In this case we cannot say that the concept of justice is a concept; according to Frege's criteria this must be rejected as ungrammatical[69]. This brings up the question of existence in his use of the logical square for propositions from Aristotle's syllogistics.

IV. Frege's predicative functions examined with the use of Aristotle's and Wittgenstein's tools

Does Frege realise the importance of the if and must key terms as used by Wittgenstein? Frege has great difficulty in dealing with a concept such as "justice" or "identity". Clearly, he cannot succeed in analysing further predicative parts of statements, without addressing the meaning of concepts, instead of their definition. Conceptual meanings, however, scarcely follow the linguistic behaviour or the linguistic interpretation of certain predicates; meanings and predicates do not always match with each other in public language[70].

Wittgenstein's radical empiricism is incompatible with Frege's analysis of meaning, which cannot uncover the historical meaning of a philosophical statement[71]. Identical results regarding truth-values derive from logical operations on elementary propositions[72], Wittgenstein states. These results themselves become the structure of these propositions. These operations then are not genuine functional operations. Proper standards depend on known facts which we hold to be relevant in the field of science and in the world. It is not a matter merely of challenging the

66 Linnebo 2018, 119.
67 Frege 1893, § 147; Frege 2013, II, §§ 144-157, 178-201.
68 Linnebo 2018, 117, n. 19.
69 Pivčević 1975, 273.
70 Pivčević 1975, 273-274.
71 Dummett 1981b, 527.
72 Hintikka 1989, 101.

wisdom of generations, or the unlikelihood of their being right. There are some elements of rationality which are trans-historical: 2+2=4 is undoubtedly valid in every society[73]. It is simply a matter of whether there can be any other criteria of rightness and wrongness than the ordinary ones for what is habitually accepted[74]. Certainly, we *cannot* and *need not* question them. The real discovery, Wittgenstein says, is that which enables us to abandon the study of philosophy at will: the decision that gives philosophy peace[75]. To pursue truth in abstraction from the circumstances under which we can realise it is destructive[76]. Ayer says that a test serves no purpose unless we are disposed to accept what it appears to show[77]. This is exactly what Aristotle believes. Wittgenstein states in *Tractatus*[78] that there are no "logical objects" or "logical constants" in Frege's and Russell's sense. This is why, he continues, the effects of truth-operations on truth-functions are always identical; the truth-function of elementary propositions is one and the same. Importantly for both Frege and Wittgenstein "logical ties" or "copulas" do not constitute links of names of particulars in elementary sentences[79]. Instead, names of objects match different logical types that complement each other like links in a chain[80]. This logical connection, Wittgenstein avers[81], is not a relationship; it solely enables the creation of a relationship[82]. In Dummett's account of Frege, however[83], to say that sentences mean the same (though they differ logico-syntactically) implies enabling explanations that require the content to be remodelled. He matches this with the use of appropriate abstraction principles necessary for this remodelling, thus overlooking Wittgenstein's warnings about both incomplete meanings and incomplete linguistics.

Wittgenstein resists reductionism. One particular cannot be combined with another particular in the same way that a property can be combined with it[84]. This is a logical impossibility[85] for Wittgenstein, as it does not take into consideration the status of those objects *qua* objects. The opposite is true for Aristotle who teaches us repeatedly, especially in 73 b27-30, that *a per se attribute is identical with that which belongs to its subject qua itself* (τὸ καθ᾽ αὑτὸ δὲ καὶ ᾗ αὐτὸ ταὐτόν). This equation of *per se* with *qua* is, paradoxically, the key to understanding Aristotle's idea of *the debased essence*, which concerns the "whatness" of individual substance. Why can certain properties be considered basic? To this question Aristotle answers with

73 Giouli 2012b, 102.
74 Giouli 2012b, 139-140 and 202. Burke 1995, 46-47.
75 Giouli 2012b, 232, n.83.
76 Burke 1983, 209.
77 Ayer 1975, 79.
78 Wittgenstein 1961, 5.4 ff.
79 Hintikka and Hintikka 1989, 92.
80 Wittgenstein 1961, 2. 03.
81 Wittgenstein 1979, 26.
82 Wittgenstein 1979, 92. Dummett 1981b, 338.
83 Linnebo 2018, 77, n. 2.
84 Wittgenstein 1961, 3.333.
85 Hintikka and Hintikka 1989, 40.

his "craftsmanship" model of the world and its history[86]. For Wittgenstein the identity of predicates with objects belongs in the realm of the logically impossible. Frege reduces this realm to the known; hence, he is unable to solve the problem by speaking of "unsaturated" predicative parts of statements (e.g., "has brown hair" or "loves") and "saturated" entities, complete in themselves, as are proper nouns[87]. This solution is arrived at through the mind: *how* a term is meaningfully used and signified (or co-used and co-signified) needs the user's knowledge and understanding[88]. For example, we have to decide when one and the same singular term refers to the same individual in all possible worlds, in arguments of a certain kind[89]. Are these individuals the same or different? Frege cannot answer this question[90], despite our modern logician's certainty that singular reference can be achieved by providing a criterion of identity for the would-be referent[91].

Hence, predicates and objects are not identical with one another: to say that something exists means that a certain predicate is satisfied, no matter whether this predicate stands for anything[92]. This remark introduces us to Frege's core concept[93] of the incommunicability of thought (shared indeed by Wittgenstein, though he understands it very differently). The subject of semantics, which concerns the connection between word and meaning, is not a valid guide to meaning for Frege. According to the Fregean distinction[94] between reference (object meant) and meaning (sense, directedness) that enables us to refer to the object, the meaning of expressions occurs in intensional contexts. For example, in order to define the concept of justice, we must refer to its extension. We cannot extend the concept without using the word 'justice' itself as meaningful. Thus, his idea of the extension of a concept (a special kind of object, as he calls it) is valid[95] within an existential context. What is the role played by the object in semantics? Objects linked by designating expressions and proper names fall into ontological categories closely related to semantic notions[96] Frege's ideas on extensional language, such as truth-functional definitions of propositional connectives, the meaning of quantifiers etc., are thus relegated to the realm of indirect informal explanation[97]. Once we have reached a definition framed wholly in terms of expressions that may be claimed to be indefinable in an absolute sense, we can consider the work of logical analysis as at an end. Frege[98], however, does not suggest the existence of a system of

86 Giouli 2012b, 135-260.
87 Pivčević 1975, 272-273.
88 Charles 2000, 160, nn. 49-50.
89 Hintikka 1975, 90.
90 Hintikka 1975, 125.
91 Crivelli 1996, 147-159.
92 Ayer 1973, 204. Charles 2000, n. 64.
93 Dummett 1981b, 122.
94 Linnebo 2018, 23, n. 4.
95 Pivčević 1975, 271-272.
96 Linnebo 2018, 136, n. 5
97 Hintikka and Hintikka 1989, 2.
98 Frege 1903, 319-324 and 368-375.

definitions of which each primitive term is absolutely indefinable. We can thus arrive at the logically complex regarding undefinable basic logical elements. This is not the same as Wittgenstein's belief that the result of an ultimate analysis is not expressible[99]. Concepts which cannot be clearly defined can only be of value in public language; not in private, Frege stresses[100]. Here the mental can be dismissed by Wittgenstein, not as the physical, but in order for objectivity to be safeguarded[101]. Aristotle states that a definitional account of the world is possible (but not of its meaning). The issues raised in the process of definition illustrate the tension between the definiens and the definiendum[102] and between the definiens and the whole definition[103].

The evolution of words has been illustrated in art. Charles Sandison (1969-), a Scottish visual artist, regards this evolutionary process as if it were taking place within a platonic cave[104]. This process concerns the *materialisation* of meanings in predication: in Sandison's work, this is represented by digital dots that hint at a 'conditional' discontinuity in the 'unconditional' continuity of the process of attribution of meaning to words. In Aristotle, however, our understanding of terms – in opposition to Frege's belief – is not necessarily sufficient to enable us to distinguish the "possessor" of the term from all other objects – real and imaginary – which the term may signify[105].

The question that arises here regarding Frege's ideas[106] is whether this "unsaturatedness" (incompleteness) of properties (predicates) of particulars can represent concepts in statements. Asserting something about a concept does not necessarily imply asserting something about an "unsaturated" function of these properties. Linking concepts with 'unsaturated' functions (that contain a "gap" in the form of a variable, such as, for example the square root of x) and objects with proper names, containing no gaps as part of their structure, does not answer the question of the use of the "unsaturatedness" of concepts. The result of the misuse here is confusion between concepts and objects: concepts become objects, Frege avers.

It is possible to agree with Wittgenstein's challenges and suspicions that any analytic truths, as also happens with inductive references, mean "the same" in all relevant respects, as it is only the criterion of relevance that can make a difference in time[107]. Wittgenstein, in *Tractatus*, espouses anti-realism but not in the form according to which the realism/non-realism argument functions. His version of anti-realism differs from the standard argument as regards a similar programme in pursuit of negating-fulfilling the assertion conditions in semantics.

99 Dummett 1981b, 256-257.
100 Dummett 1981b, 190.
101 Dummett 1981b, 487.
102 Charles 2010, 135, 137-138.
103 Charles 2010, 17.
104 https://sandison.fi.
105 Charles 2000, 105ff, 161-165.
106 Pivčević 1975, 272-273.
107 Burke 1988, 211.

Belief in the ideal concept of the world uses conceptual equipment to identify events. It frames statements different from those framed by scientists and secular historians. They both thus do more than just disagree over the truth-values of the same statements, as these statements are too far apart to contradict each other or even to be compared. This is precisely the case with Aristotle, who suggests that we should attempt to realise the ideal of truth – instead of trying to perceive order and reality at the start of our exploration. We must be very careful not to undercut the standard argument of realism/non-realism in Aristotle. We can see it at work as regards what merely happens – not what might happen. However, according to Aristotle, we shall always lack the ideal concept of the world as it is in itself[108]. Wittgenstein, Hacker claims, propounded no theory of meaning of this kind, having good reasons for eschewing such a programme[109]. Thus, Wittgenstein argues that an elementary proposition does not have any truth-conditions[110], stressing thus only the criteria of relevance as regards its soundness. Hence, an elementary proposition no more has truth-conditions than does a tautology or a contradiction[111]. The way people experience reality varies by the moment, and thus is transitory (as is a postulate). The kind of demarcation which expresses the aforementioned tension between what is real and what is not real is very accurately stressed by Wittgenstein; we come to know the unreliability of perception: "this is not how it is. Yet this is how it has to be!", i.e., this has to be our attachment to the realisation of the ideal of truth[112]. Indeed, Hacker claims[113] that Carnap rightly rejected the *Tractatus* conception of word-meaning. Speaking of word-meaning, Carnap is not speaking of entities that are part of the substance of all possible worlds. He claims that in the pseudo-proposition "Caesar is a prime number" both "Caesar" and "is a prime number" have a meaning, although their combination is meaningless: they do not stand for real entities which have meanings. Because the necessary and sufficient condition for a word to be meaningful, according to Carnap, are (i) that its syntax is determined, (ii) that its deducibility relations are fixed and (iii) that the truth-conditions of elementary sentences in which it is embedded are settled. So, Hacker continues, Carnap accepts a version of the context principle, albeit with a different rationale from those offered by Frege and Wittgenstein. In Carnap's account, both "Caesar" and "a prime number" are meaningful, i.e. the rules in their use in sentences have been stipulated. But these rules preclude the combination "Caesar is a prime number" in as much as the rules for the use of this predicate preclude its occurrence except as a predicate of numbers. These rules preclude this combination in as much as the rules for the use of the proper name exclude its occurrence as the subject term for such predicates. *Tractatus*,

108 Giouli 2022b, 268.
109 Hacker 1996, 242.
110 Hacker 1996, 279.
111 Hacker 1996, 52-53.
112 Giouli 2012b, 164-165 and n. 43.
113 Hacker 2003, 17-18.

hence, Hacker concludes[114] is not concerned with the application of logic. Again, how then are the relevant mythological terms and terms of historical values actually used?

Is this inconsistent with Frege's criteria as regards his use of Aristotle's square of oppositions for categorical statements regarding syllogistics? If the concept of justice, for example, is no longer a concept, but is converted into or represented by an object, this is inconsistent with his philosophy of functions whose values are always truth values. A thought becomes the content of thought and does not become the content of thought at the same time; something absurd and ungrammatical ("the concept of justice is a concept" *is ungrammatical* according to Frege's own criteria). That was what led him to the limits of logical analysis, as mentioned above, as the ungrammatical applies to concepts and the truth- values they express; not to objects.

Frege has understood the importance of the *if* and *must* key terms as regard the difference between what merely happens and what could happen. The generalization which regards the values of functions, and the value-ranges of an equation is not provable[115]. This generalisation (if true, we add) must be taken as a basic logical law. Otherwise, Frege states, it cannot be proved. Here, we see how he hints at the key terms suggested above, and how he tackles Aristotle's idea of the existential import that the square of opposition inevitably carries. Certainly, a non-recognisable concept of the ideal (of the unknown) can neither negate nor confirm such laws, as they lie in different conceptual worlds. Frege, however, hastens to take the opportunity to stress the ideal character of mathematics. The notion of a function is logically prior to that of a value-range, he adds. This fundamental difference, however, haunts him, as he continues to doubt the omnipotence of mathematical reasoning. Hence, he maintains that the universal quantification of an equation between the values of functions, on the one hand, expresses the same sense as the equation between their value-ranges, on the other. He stresses that this sense, though the same, is *differently* (we underline) expressed. He offers[116] the example of two parallel lines in their parallel directions, in order to clarify further that sameness[117]. But he stops at this point, never commenting on the difference between these two conceptual worlds. His almost total silence as regards the contradictory impact brought by the ideal on his use of the square of logic is embarrassing. He never speaks of the transition from one method of expressing a thought to another method of expressing the same thought. This transition is called an indispensable transformation,[118] in the form of a law of logic[119]. We understand how intolerable it is for Frege not to be able to prove the existence of such laws and transitions. As Dummett

114 Hacker 2003, 21.
115 Frege 1891, 9-10.
116 Frege 1891, 11.
117 Frege 1894, §64.
118 Frege 1953, §9.
119 Frege 1953, §§146-147.

puts it[120], this turmoil (caused among other things by Russell's critique, as we have seen above) has brought on a crisis as regards both the analytic character of arithmetical truth and identity and differences of thoughts as regards their analysis and their deconstruction. But is not this crisis present in Frege's works? The similarity or difference of the truth values of the statements identifying facts does not really concern him. Is it, perhaps, the difference between those statements which establish what really is and the statements that aim to identify the possible, which is his major concern? These two types of statements cannot contradict each other, as they are too far apart. They exist in different conceptual worlds and are thus determined by different conceptual ranges. Aristotle and Wittgenstein pursue the development of conceptual armament from the abstract to the concrete; from what merely happens to what might happen. Is this what Frege does?

Values of functions do not work out for variables (numbers) and arguments. How to refer to a concept remains an impasse in Frege[121]; unless this is accomplished within a context that bears the existential import for the square. It is impossible to do so unless we consider the concept an object; otherwise, we have to express concepts ad infinitum by predicative expressions. We do not know, however, according to Aristotle, how we come to grasp terms for kinds. Frege finds this the most mysterious question of all[122]. The process of grasping a thought is *mental* in character. Therefore, according to Frege, it need bother us no longer in logic.

Philosophers have different opinions on whether, when asserting something about a concept, one is talking of an "unsaturated" predicative functional expression. Daniel Rothschild has examined[123], using the Frege-Geach tools, the reasons why ascriptions of graded judgements imply appropriately modelling imprecise beliefs. This modelling, however, is arbitrarily used. Order and reality cannot be perceived from the outset. Dismissing all claims that we can perceive them from the outset means that we are attempting to realise the ideal of order while remodelling it.

Sentences, then, and nouns referring to truth-values, in fact refer to two mysterious objects, the true and the false[124]. Any attempt to formulate facts and evidence in our field of interest presupposes that we have already adopted a range of concepts that determines the identification of facts and the pursuit of values. We cannot adopt a standpoint independent of this range in our pursuit of the proper range. Proper standards depend on what we regard as relevant in the already known in the field of science and the world in general. Ayer [125] adds that the reason why the sentence 'Mary Ann Evans might not have been Mary Ann Evans' is non-sense is not that we consider the possibility of denying a necessary

120 Dummett 1981b, 335-336, esp. 336.
121 Pivčević 1975, 272-273.
122 Frege 1969a, 157.
123 Rothschild 2012, 99-114.
124 Ayer 1982, 265-267.
125 Ayer 1982, 267.

truth but that we *lack* precise criteria to fix the reference of either occurrence of the proper name. This shows Frege's declining rationality in his attempt to sort out values of functions for variables and arguments, thus opting for the impossible.

Language as a social phenomenon is a game for many players, and in order to take part in it we have to accept rules that are not of our own making[126]. This includes common standards concerning what may or may not be done and said. To leave everything as it is – tacitly admitting our inability to utter the literal meanings of our propositions – is the only possible and proper use of language games, according to Wittgenstein[127], who believes that we should not interfere with the actual use of language. Therefore, its description should constitute the sole aim of our enquires. For Aristotle, engagement with instances of reality will enable us to grasp terms of kinds. We do not know, however, according to Aristotle, how we come to grasp them. Frege's reductionist theory, on the other hand, is purely idealistic. He holds that thoughts are often not consciously formed and, further, that what he calls objective is not constituted by our minds or by our thinking of it. This is a sheer contradiction however, in view of his claims that non-material objects exist[128]. Hence, talking about matter is really just talking about mind; reality is a complete mystery. This connects neither with Wittgenstein, nor with Aristotle, both of whom admit that we need (even frustratingly) to explain reality in terms of something that itself needs no explanation. If we consider that y is caused by x and x is the cause of y, thus y is caused by the cause of y. We have achieved no more than tautology. This does not function as an explanation. Aristotle and Wittgenstein acknowledge that over and above explanation of the world there is something more[129]. which does not fall within our secular ways of thought, within our conceptual range, and must be left unexplained. It cannot be conceptualised or articulated. It cannot stand for anything we are constrained by reason to accept[130]. But does Frege postulate Descartes' invisible, soundless ghost with no manifestations beyond the puzzling phenomenon of y in order to permit worthless tautological explanations and acknowledge by the constraint of reason that y stands for certain features of the world and of our experience of it?[131] Definitely not. Explaining everything according to Frege's model means that these explanations are worthless because he has taken them to refer to statements of problems which we are able to answer through the use of our intellect[132]. Had Frege understood that we cannot explain it in terms of something that itself needs no explanation? Plainly, no further inferential steps are needed to identify an explanation in terms of something which does not itself require to be explained. If there were evidence as regards belief and summoning of the inexplicable,

126 Burke 1995, 45-47.
127 Wittgenstein 1953, I, 124.
128 Cobb-Stevens 1990, 48, n. 2.
129 Burke 1995, 65.
130 Burke 1995, 72.
131 Burke 1995, 73.
132 Burke 2000, 87-88.

Wittgenstein adds, this would in fact destroy the whole business[133]. That is why Wittgenstein claims that the real discovery, as regards feelings and impressions of the world is that which makes us capable of stopping philosophising at will. This discovery leaves philosophy in peace, so that it is no longer tormented by questions which bring *itself* in question[134].

It is true that we grasp terms, but we do not know how. We grasp universals but we are unable to understand how we do so, according to Aristotle. Hence, we cannot shift any necessary modal operator[135] forward to infer that our knowledge carries some infallible logical guarantee of success. All that this success can concern is fact. Inversely, for Frege[136] a logic that allows modal operators to function over indefinitely extensible domains is a purely intuitionistic one. Indeed, this is the only way to make sense of the indefinite extensibility of a concept in Frege's sense, a possible way to define yet further instances of the concept[137]. Does Dummett's idea of extensibility fail to make a case for this specific plurality? Dummett only refers to larger and larger hierarchies/totalities (definitely formed, however) all of whose components form part of the totalities. Deprived of meaning, however, mathematics cannot tell us what really is in the world, apart from intuitions; and neither can logic.

It was this failure which was found intolerable by Frege – not Russel's attack on his ideas that proof of existence cannot be coupled with the use of a proper noun[138]. Avoiding the contradiction innate in the use of a noun presupposes means that a proper noun cannot be expanded into a propositional function. Then we must place restrictions on our functions if we wish to avoid contradictions. Predicating of a class that it is or is not a member of itself is a catalyst for this contradiction and for the use of the logical square. It is reasonable to say, on the one hand, that the class of things which can be counted is itself something that can be counted. On the other hand, the class of man is not itself a man. We have two types of classes: the type of classes which are members of themselves and the type of classes which are not members of themselves. If we ask whether this second type of classes is a member of itself, we get the contradictory answer that if it is, it is not, and if it is not, it is[139]. We cannot significantly attribute any property to propositions in general but only to propositions of such and such an order, say, n. This itself being a proposition of a higher order than n, no paradox arises[140].

Nobody can deny[141] that Frege stresses that a proper noun stands for that sort of object which we intuitively take to be its possessor. Hence two proper nouns would have the same semantic value, if they had the same bearer. The existence

133 Wittgenstein 1966, 56.
134 Wittgenstein 1953, I, 133.
135 Giouli 2012b, 126. Cottingham 1984, 20.
136 Linnebo 2018, 73, n. 41.
137 Linnebo 2018, 69, n. 37.
138 Ayer 1973, 54.
139 Ayer 1973, 44.
140 Ayer 1973, 45.
141 Dummett 1981b, 159-160.

of intensional contexts here does not have any impact on the semantic value. But does it not? Russell has shown the opposite. Frege[142] takes refuge in his *Bedeutung* (reference, object meant) concept of the noun which is its bearer. This name/bearer relationship acquired at the start is not affected by any ordinary *Bedeutung* acquired in an intensional context afterwards. This does not invalidate Russell's belief in the existential import and restrictions of contradictory statements. It only shows that Frege was reluctantly able to understand the way out of the impasse of the contradictory: there can be no other criteria than ordinary ones for what is habitually done. That was exactly the way suggested by Aristotle's use of the "square of oppositions". Frege thus paves the way for a philosophical understanding of Aristotle's and Wittgenstein's anti-realistic accounts of the world. Charles[143] stresses the fact that many will find this intolerable, as it conflicts with the strongly realist tenor in Aristotle. This means that we cannot have any conception of what a man is without having some means of referring to particular men. This straightforward equation of the semantic value of a name with its bearer, however, has created the problem emphasised by Russell.

It is the bearer (a poppy, for example) of, say, "redness" that can suggest the answer to what it is to be red; not the poppy *per se*[144]. This does not mean that the red poppy is not there at all, because it exists conditionally. One can respond to Wittgenstein's challenges and suspicions that any analytic truths, as also happens with inductive references, mean "the same" in all relevant respects, as it is only our criterion of relevance that can make the difference in time[145].

V. Concluding Remarks

We are unwilling to accept either that science undertakes the logically impossible or that such science cited as a paradigm example of rationality has an elementary logical fallacy built into its essential method of argument. We cannot establish the truth of unrestricted generalisations by empirical means. Aristotle, as we see, is led to such a disconcerting, counter-intuitive conclusion offering his own idea of what science is[146]. To furnish sufficient inductive means then, according to Aristotle, we have to supplement them with the inexplicable, with concepts from the realm of the unknown[147]. Whether the conditions for determining truth values are fulfilled or not, there remains the question of the realist/non realist argument in how we understand terms and semantics for the said problems. Hence, all questioning as regards the pursuit of the logically impossible ceases to be cryptic and opaque with Aristotle's account of the world. This is a differing account than the one

142 Frege 1892, 25-50.
143 Charles 1997, 241.
144 Burke 2000, 36.
145 Burke 1988, 211.
146 Giouli 2012b, 132, n. 9.
147 Giouli 2012b, 155, n. 32.

proposed by David Charles: Aristotle's idea of the logically impossible as herein stressed versus Charles' interpretation of Aristotle's need to proceed no further. This means that Aristotle could not proceed in terms of something which does not itself require to be explained[148]. Charles, thus, stresses the interconnection between nature and explanation.

A less demanding form of realism foreshadowing, say, Kant's distinction between phenomena and noumena[149], would favour Frege's ideas. Aristotle's advice as regards this form rejects all forms of robust realism that led Frege into an impasse. Frege's most remarkable achievement in the history of ideas is his rationalistic zeal in his fight against psychology. This achievement, however, is brought to a standstill by logical analysis, thus suggesting his almost total silence as regards the predicative functions of concepts and meanings. Is this where his analysis started from? Nobody can answer this. Frege has laid the way for a philosophical understanding of Aristotle's and Wittgenstein's views of the world. Their views did not just concern another consistent system of reference, Frege believed. Prior moves in logic we have made regarding the recurrent moves would certainly form a limit to this system[150]. The account of the world given by both Aristotle and Wittgenstein opts for the logical impossible as an anti-realistic account of it. Frege, on the other hand, understood with a reluctant heart that we cannot discover reality through the use of an omnipotent reason. The decision to opt for the logical impossible seems to be intolerable for him. At least, Frege has called our attention to the limits of what can be answered, using the idea of the inexplicable in Wittgenstein's sense of anti-realism. Aristotle and Wittgenstein teach us to persist in a vain attempt to impose order and laws on thought; by employing the aristotelian "craftsmanship" model of the world, on the one hand, and by taking refuge in the wittgensteinian notion of silence, on the other[151]. Unless we consider what must or cannot happen in all sets of circumstances, mathematics does not tell us *whether or not a modern logician's equinumerical collection exists in the world*, nor prove her/his suggestion that objects are possible referents. S/He avers that, if something is an equinumerical collection (of knives and forks) these two must be, as a matter of logical necessity, identical in number. S/he overlooks the *if* and *must* key terms[152] in this proposition which is purely *hypothetical* in form. Still, a coherent account of the world that does not conform to what is logically necessary *is* yet possible. This is what science has taught us thus far, *not* pure mathematics. After all, the impossible does not emerge from the possible[153]. Deprived of meaning, mathematics cannot tell us what really exists in the world; apart from the logically necessary which is true in all possible worlds; neither can logic, as far as facts are concerned. Frege within his square of logic has well understood that the ideal apart from being self-contradictory does

148 Charles 2000, 255-264.
149 Moutsopoulos 1975, 285-290.
150 Ayer 1975, 79-92.
151 Charles 2003, 103-126.
152 Burke 2000, 16.
153 Giouli 2012b, 109, 160 and n. 39 and 238.

not fall within the range of explanatory concepts, in the form of a recognisable concept. Frege does not offer another explanatory range of concepts, and thus tacitly acknowledges the inexplicable.

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