

EPISTEMIC SOPHISMS, *CALCULATORES* AND JOHN MAIR'S CIRCLE

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Abstract: This paper focuses on the early sixteenth-century epistemic logic developed by John Mair's circle and discusses iterated epistemic modalities, epistemic closure and Bradwardinian semantics related to the logic of epistemic statements. These topics are addressed as part of setting up and solving epistemic sophisms based on traditional scenarios which can be traced back to fourteenth-century British epistemic logic. While the ultimate source for the debate appears to be the second chapter of William Heytesbury's *Regule solvendi sophismata*, the immediate source is the Italian editorial, commentarial and philosophical tradition, notably Paul of Venice and Cajetan of Thiene.

Keywords: Oxford Calculators; John Mair's circle; epistemic logic; epistemic modalities; sophisms; scholastic logic.

1. Introduction

Around 1330 the solution to epistemic sophisms grew into a sovereign genre, commonly referred to as '*De scire*' or '*De scire et dubitare*', through the works of authors such as William Heytesbury, whose treatise would influence epistemic logic all the way to the mid-sixteenth century. Heytesbury's logic of believing, knowing and doubting is predominately based on the Aristotelian distinction between composed and divided sense, which captures different readings of doxastic and epistemic statements and relevant inference rules.

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His treatise *De scire et dubitare* from *Regule solvendi sophismata* consists of a series of epistemic sophisms, the doctrinal core discussing logical independence of *de re* and *de dicto* contexts and iterated modalities, and the solution to the opening sophisms. All sophisms are developed as games of *obligationes*, i.e., as disputations based on a posited scenario, where the key issue is the compatibility of knowledge and doubt.¹ These scenarios would later become widely used and, quite possibly, Heytesbury himself inherited some of them from his forebears (such as Richard Kilvington).² Together with explicit references, included in the main text or added in the margins, the common practice of reusing the scenarios of epistemic sophisms and the common form of such sophisms makes both the continuity with and all deviations from the original sources relatively easy to detect.

This paper focuses on the development of the fourteenth-century tradition of British epistemic logic in the sixteenth-century John Mair's circle in Paris. The primary corpus includes texts by Jerome Pardo (d. 1502), a teacher and collaborator of John Mair, John Mair (1467–1550), Mair's student Antonio Coronel (d. around 1521), Gaspar Lax (1487–1560), who was a student of John Mair and a teacher of Juan de Celaya, and Juan de Celaya (ca. 1490–1558), a student of Jan Dullaert, Gaspar Lax and possibly John Mair, and a teacher of Domingo de Soto.³ Three groups of problems will be addressed: iterated epis-

1 See WILLIAM HEYTESBURY 1494(1) (the critical edition is not available, for an English translation see WILLIAM HEYTESBURY 1988), WILLIAM HEYTESBURY 1494(2) analysed in STUMP 1989, BOH 1993, HANKE 20018(1), HANKE forth.(1), and HANKE 2021 (containing a critical edition of Pseudo-Heytesbury's *Casus obligationis*). For a basic overview of *obligationes*, see SPADE, YRJÖNSUURI 2020, for *obligationes* in the circle of the Oxford Calculators, see YRJÖNSUURI 1990. Finally, there is ARRIBAS 1993, which is relevant to the genre of *obligationes* in John Mair's circle. The term 'scenario' as a translation of 'casus' or 'positum', which means the same in this context, is from READ 2020(1).

2 See KILVINGTON 1990, 119–151, analysed in (among others) STUMP 1989, 222–231 and BOH 1993, 62–77.

3 For biographical and bibliographical data see the following footnote. The quotations rely on working transcriptions; the orthography and punctuation has been adapted,

temic modalities pertaining to the debate on whether one can doubt one's own knowledge; epistemic closure pertaining to epistemological debates; and Bradwardinian sentential semantics. The body of epistemic sophisms introduced by the aforementioned authors is by no means limited to those discussed below, but these in particular are interesting even from the modern perspective, as a consequence of being directly relevant to the formal properties of epistemic modalities, and can be motivated in an intuitive way. The study is rooted in a growing body of research into three related issues, namely post-medieval logic in general,⁴ scholastic doxastic and epistemic logic,⁵ and epistemology in John Mair's circle⁶. It is intended to contribute to each of these fields to some extent.

2. Iterated Epistemic Modalities

Iterated epistemic modalities were a standard issue of fourteenth-century epistemic logic and were comprehensively researched by Ivan Boh.⁷ Two instances of debating these topics by Jerome Pardo and Gaspar Lax will now be

rare and minor corrections are limited to obvious errors and are not indicated.

4 The seminal work in the field is ASHWORTH 1974(2); for more recent overviews by the same author, ASHWORTH 2008 and ASHWORTH 2016. For an introductory publication which covers the authors pertaining to John Mair's circle, see BROADIE 1987, BOH 2001 or LAGERLUND 2017. As for the more specific material, Alexander Broadie authored multiple publications on John Mair's circle, including BROADIE 1983 and BROADIE 1985; for bibliography, see DURKAN 1950, LOHR 1975 and LOHR 1978.

5 The most general sources are BOH 1993 and BOH 1997. For epistemic logic analysed as related to *sophismata* and *obligationes*, see STUMP 1989, 215–249, YRJÖNSUURI, COPOCK 2016, 277–280, and HANKE 20018(1), 2018(2), forth.(1) and forth.(2).

6 See BROADIE 1993 and BROADIE 1995, KARGER 2009, LAGERLUND 2019, LONGEWAY 2009.

7 See BOH 1993, 73–76 and 111–112, and BOH 1984 which focus on William Heytesbury, Peter of Mantua, and Cajetan of Thiene. Other sources pertaining to this tradition, notably John Wyclif, John of Holland, Paul of Venice, Paul of Pergula, and Mengho Bianchelli were analysed in HANKE 20018(1) and HANKE 2018(2). There is still a large number of unanalysed relevant texts, such as JOHN HUNTER 1999, 418–445, presumably one of Paul of Venice's sources, and a number of unedited treatises related to Heytesbury's treatise (which could turn out interesting despite their relatively minor influence).

presented. Despite their respective specific contexts, they display notable fourteenth-century influences.

2.1 Jerome Pardo: *Medulla dyalectices* (1500/1505)

Pardo addresses iterated epistemic modalities in the seventh chapter of his *Medulla dyalectices*. The chapter addresses the truth conditions of statements as related to the appellation of terms, including ‘*appellatio rationis*’ in epistemic statements; note the underlying use of the terminist semantics.⁸ As part of a discussion of relevant logical rules, Pardo introduces the axiom **K** of epistemic logic: knowledge distributes over implication or, in the original phrasing, if an inference is valid and an agent knows that it is valid and knows that the antecedent holds, then the agent also knows that the consequent holds. Pardo notes here that the rule might include an additional requirement that the agent is not diverted from contemplating the problem, but ultimately does not regard such extension as quite necessary. Despite the terminist phrasing, the principle is viewed as pertaining to the genre of consequences.⁹ Afterwards, Pardo introduces a *dubium* whether the same statement can simultaneously be a matter of knowledge and doubt or conjecture and gives a negative answer, followed by counterexamples and their analysis.¹⁰

8 For the widely discussed notion of *appellatio rationis*, see (e.g.) NUCHELMANS 1988, BOH 1993, 85–86, and the recent PANACCIO 2012; for *appellatio rationis* as related to logical omniscience, see HANKE forth.(1). For a general overview of terminism, see READ 2019.

9 “Supponamus ulterius pro materia deducenda aliquas regulas generales consequentiarum. Prima regula: si aliqua consequentia est bona et scita esse bona et antecedens est scitum ab aliquo (capiendo antecedens pro significato), consequens est scitum ab eodem (capiendo consequens pro significato). (...) Tamen posset dici, licet non sit necesse, quod regula predicta sic intelligitur, quod scito antecedente et scita bonitate consequentie, si voluntas non divertat intellectum a consideratione conclusionis, scitur etiam conclusio, et loquor de antecedente totali,” JEROME PARDO 1505, fol. 97ra–rb.

10 “Quibus suppositis sine ampliori declaratione quero tale dubium: utrum eadem propositio sit dubia et scita vel scita et opinata, vel magis proprie loquendo an idem significatum secundum eandem propositionem sit scitum et dubium vel scitum et opinatum.

The eighth counterexample argues that since it is possible that someone knows something while merely conjecturing that he knows it, the same thing can be known and conjectured, and, as a consequence, known and doubted.¹¹ The proof breaks down into the proof of the assumption and the proof of the inference. According to the first sub-proof, the scenario that someone knows that Socrates is running while conjecturing that he knows it is consistent. The justification dismantles one particular threat to that scenario's consistency, namely the inference to the conclusion that the agent knows that he knows that Socrates is running via the axiom of positive introspection ('if **p** knows that *X* is the case, then **p** knows that **p** knows that *X* is the case'). This principle is dismissed by pointing out the independency of direct and reflexive cognitive acts.¹² The second sub-proof argues that the scenario entails the coexistence of knowledge and conjecture with respect to the same problem. The knowledge part is presupposed in the scenario. The conjecture part follows from the assumption that the agent conjectures that he knows that Socrates is running: whoever conjectures that he knows that Socrates is running *ipso facto* conjectures that such knowledge is factually correct. That completes the ar-

Respondeo talem conclusionem ponendo: non est possibile idem significatum totale propositionis esse scitum et opinatum ab eodem, scientia et opinione precise representantibus tale significatum, et hoc sive teneatur primus modus dicendi de complexe significabilibus sive non," JEROME PARDO 1505, fol. 97rb. Note the mentioning of propositional ontology as well as its irrelevance to the problem.

11 "Octava instantia: possibile est quod opinaris te scire Socratem currere et tamen quod scias Socratem currere, quo facto sequitur quod possibile est te scire Socratem currere et opinari Socratem currere, et per consequens idem est scitum et dubium," JEROME PARDO 1505, fol. 101rb. Note that first edition, i.e., JEROME PARDO 1500, of *Medulla dyalectices* has a different numbering but the text is, in this particular regard, identical. The same is true for Pardo's text discussed below in section 4.

12 "Primum patet, videlicet quod possibile sit te scire Socratem currere et simul opinari te scire Socratem currere, quia non sequitur: 'ille scit Socratem currere, ergo scit se scire Socratem currere'. Non enim necesse est quod habito illo actu, habeatur actus reflexus. Ideo non apparet repugnantia quod scientia qua scis Socratem currere stet cum opinione qua opinaris te scire Socratem currere," JEROME PARDO 1505, fol. 101rb.

gument.¹³ The reason seems to be that knowledge breaks down into the existence and factual correctness of a mental act.¹⁴ To conjecture that I know X breaks down into conjecturing that I assent to X and to conjecturing that X is the case, presumably since conjecture is closed under conjunction elimination.

The proof of the corollary (that the same thing can be known and doubted) is not introduced explicitly. It is not a direct implication of the scenario, since conjecturing excludes rather than entails doubting.¹⁵ However, one could replace ‘to conjecture’ with ‘to doubt’ in Pardo’s argument: the scenario that someone knows that X is the case and doubts whether he knows that X is the case is consistent, whence the same thing can be simultaneously known and doubted. The proof decomposes into two sub-proofs. The first sub-proof (that the scenario is consistent) secures the consistency of the scenario by rejecting the axiom of positive introspection (which is, by assumption, the most severe problem). The second sub-proof (that the inference is valid) argues that the scenario entails that someone simultaneously knows and doubts that Socrates is running, since whoever doubts that he knows that X is the case *ipso facto* doubts that X is the case. For that inference to be legiti-

13 “Sed probatur secundum, videlicet quod ad aliquem scire Socratem currere et opinari se scire Socratem currere sequatur quod scit Socratem currere et quod opinatur Socratem currere. Nam primo habetur quod scit Socratem currere. Sed quod opinetur Socratem currere, probo, quia bene sequitur: ‘*opinatur se scire Socratem currere, ergo opinatur ita esse sicut per illam scientiam significatur*’. Et sic habetur quod scit Socratem currere et opinatur Socratem currere, quod erat probandum,” JEROME PARDO 1505, fol. 101rb–va. Note that these arguments have numerous presuppositions regarding the logic of epistemic and doxastic verbs, which makes them vulnerable to criticism.

14 “Ideo posset dici quod propositio de ‘*scio*’ exponitur per unam copulativam in qua una partium ostendet existentiam illius qualitatis, altera vero exprimet quod ita est sicut per talem qualitatem significatur, ut ista propositio: ‘*scio Socratem currere*’ posset sic exponi: ‘*hec scientia est (demonstrando illum assensum) et Socrates currit*’,” JEROME PARDO 1505, fol. 101va.

15 The reason is that conjecturing entails assenting to a sentence, whereas doubting a sentence entails neither assenting to nor dissenting from it (see JEROME PARDO 1505, fol. 96ra–va). As a result, the two acts are mutually exclusive.

mate, one has to assume that whoever doubts whether he knows that X is the case *ipso facto* doubts whether X is the case, because knowledge breaks down into etc.

Pardo summarises his solution to this counterexample in a series of theses, the first two of which will now be presented: first, it is not absolutely impossible or inconsistent that someone conjectures or doubts that he knows something; second, it is consistent to assume that someone knows something while conjecturing or doubting that he knows that.

The first thesis regards as consistent the scenario that someone conjectures that he knows something. The proof is that if the scenario did imply a contradiction, it would most likely be this one: '*p* knows that Socrates is running and *p* fails to know that Socrates is running'. However, the inference: '*p* conjectures that *p* knows that Socrates is running; therefore, *p* knows that Socrates is running' is invalid, since conjectures are not veridical. Similarly, the scenario that someone doubts that he knows something is considered consistent.¹⁶ This argument is problematic for at least two reasons. First, there are reputable medieval proofs that such a scenario is inconsistent, which makes the argument weak in terms of historical awareness.¹⁷ Second, it is meaningless to claim that a scenario is consistent unless the relevant rules of inference are

16 "Prima propositio: non est absolute impossibile quod aliquis opinetur se scire Socratem currere. Probat, quia ad hoc non sequitur aliqua contradictio, quia maxime sequeretur ista quod sciret Socratem currere et quod non sciret Socratem currere. Sed illa non sequitur, nam ego dicam quod non scit Socratem currere, unde non valet hec consequentia: '*tu opinaris te scire Socratem currere, ergo tu scis Socratem currere*' Nam possibile est quod opineris aliter quam est, ut videlicet opineris me scire Socratem currere, et tamen non sciam Socratem currere, quemadmodum faciunt presumptuosi, qui dicunt opinantes secundum unam significationem. Et per idem patet quod non est repugnantia quod aliquis dubitet se scire Socratem currere," JEROME PARDO 1505, fol. 101va.

17 For such arguments proposed by Heytesbury, John of Holland and Paul of Venice (which appear to be representative of a broader corpus), see BOH 1984, BOH 1993, 67–76 and 111–112 and HANKE 2018(1), 150–158, 164–169 and HANKE 2018(2), 214–233. Incidentally, some of the historical arguments derive a different form of contradiction.

specified and in this particular case, a self-contradiction can easily be derived from the scenario in a sufficiently strong logical system.¹⁸

The second thesis, which is presented as probable, is that the scenario that someone knows that Socrates is running while conjecturing that he knows that Socrates is running, is inconsistent, since the person would at the same time know that Socrates is running and conjecture that Socrates is running. Pardo argues that whoever conjectures that he knows that X is the case, conjecturally assents to the statement: ' X is the case and the knowledge (that X is the case) exists' implicitly (*virtualiter*), as this conjunction is the *explicans* of epistemic statements.¹⁹ Also, whoever assents to a conjunction conjecturally, *ipso facto* assents to its sub-formulas conjecturally. The function of virtual con-

18 The scholastic arguments aside, let us outline such argument in the axiomatic system S5 (as defined in, e.g., HUGHES, CRESSWELL 1996, 51-70), validating both positive and negative introspection, i.e., both 'if p knows that X is the case, then p knows that p knows that X is the case' and 'if p fails to know that X is the case, then p knows that p fails to know that X is the case'. Furthermore, let us assume that if someone doubts that X is the case, then he does not know whether X is the case, i.e., he fails to know that X is the case and he fails to know that X is not the case. Now let us assume as a hypothesis that p doubts whether he knows that X is the case. As a result, p fails to know that he knows that X is the case and p fails to know that he fails to know that X is the case. In that case, either p knows that X is the case, or he fails to know that X is the case (a tautology). That splits the scenario into two sub-hypotheses. First, the hypothesis that p knows that X is the case: if p knows that X is the case, then p knows that he knows that X is the case (by positive introspection). However, the scenario entails that p fails to know that he knows that X is the case (see above). A contradiction. Second, the hypothesis that p fails to know that X is the case: if p fails to know that X is the case, then p knows that he fails to know that X is the case (by negative introspection). However, the scenario entails that p fails to know that he fails to know that X is the case (see above). A contradiction. As both mutually exclusive hypotheses exhaustively develop the original scenario, that is self-contradictory (since it is self-contradictory on both sub-hypotheses).

19 "Pro solutione advertendum est quod ille terminus '*scientia*' supponit pro quadam qualitate in anima existente connotando quod ita sit sicut per ipsam significatur (...) Ideo posset dici quod propositio de scio exponitur per unam copulativam in qua una partium ostendet existentiam illius qualitatis, altera vero exprimet quod ita est sicut per talem qualitatem significatur, ut ista propositio: '*scio Socratem currere*' posset sic exponi: '*hec scientia est (demonstrando illum assensum) et Socrates currit*'. Ex quo patet quod assentire se scire Socratem currere est assentire illi copulative: '*hec scientia est et Socrates currit*,'" JEROME PARDO 1505, fol. 101va.

jecture in this argument suggests that conjectural assent is closed under entailment.²⁰

As an alternative, Pardo contemplates the position ascribed to an anonymous *doctor*, who claims that one's will can force a dissent to the law of non-contradiction or an assent to a self-contradiction. Similarly, someone can assent to a conjunction without assenting to its sub-formulas.²¹ Pardo ultimately considers his own view more probable and offers two restatements thereof. First, he replaces 'conjecturing' with 'doubting': it is impossible to know that Socrates is running while doubting that knowledge. Second, it is impossible to have evidence for a statement while doubting that one is in possession of such evidence.²² Pardo does not elaborate, but he could be open to restating the proof of the second thesis as follows: doubting that one knows that X is the case entails doubting that X is the case, which makes the scenario inconsistent. The most controversial step is, again, the assumption that doubt is closed under entailment (or, at the very least, under conjunction elimination), together with assuming that knowledge is veridical (the axiom

20 "Secunda propositio probabilis: non stat quod aliquis sciat Socratem currere et opinetur se scire Socratem currere. Probat, nam ex illo sequitur quod ille scit Socratem currere et opinatur Socratem currere. Nam quod ille scit Socratem currere patet ex supposito. Sed quod ille opinetur Socratem currere patet, quia opinari se scire Socratem currere est opinari virtualiter istam propositionem: '*Socrates currit et hec scientia est*'. Qui autem opinatur aliquam copulativam, opinatur quamlibet et eius partem, ergo ille opinatur quod Socrates currit," JEROME PARDO 1505, fol. 101va.

21 "Oppositum tamen istius propositionis potest defendi proterve maxime si teneatur opinio unius doctoris quod voluntas prave affectata cum aliquibus motivis potest dissentire primo principio, ut puta quod voluntas posset facere quod intellectus unico assensu assentiret isti copulative: '*Socrates currit et Socrates non currit*', nulli tamen parti. Oporteret enim assentire assensu proprio illi parti. Et non minus videtur inconveniens quod quis opinetur istam copulativam: '*hec scientia est et Socrates currit*' et non partes seorsum," JEROME PARDO 1505, fol. 101va.

22 "Tamen probabilius est tenere illam secundam propositionem quam eius oppositum, per quam patet etiam quod hec copulativa est impossibilis: '*aliquis scit Socratem currere et dubitat se scire Socratem currere*'. Et consimiliter impossibile est quod aliquis habeat evidentiam de aliqua propositione et dubitet se habere evidentiam de illa propositione. Patet ex dictis satis," JEROME PARDO 1505, fol. 101va.

T). Also, note that the axiom of positive introspection is not cited explicitly.²³

As a counterargument to the second thesis, Pardo posits the scenario that someone knows that Socrates is running based on three pieces of evidence while doubting that such evidence is sufficient, and hence doubting that he truly knows that Socrates is running. Pardo elaborates on the set-up by emphasising that the agent knows that Socrates is running (presumably since the evidence is, in fact, sufficient) and even contemplates whether he knows that, but fails to know that he knows that Socrates is running.²⁴ The reason why Pardo ultimately dismisses the counterargument lies in the concept of evidence-based knowledge (*scire per evidentiam*): to doubt whether a statement is sufficiently supported by a piece of evidence is incompatible with the very notion of evidence. If evidence for a statement did not warrant it automatically, it would not be evidence in the first place.²⁵

23 A proof of the same thesis based on positive introspection might go as follows: if **p** doubts whether he knows that X is the case, then **p** fails to know that he knows that X is the case. However, if **p** fails to know that he knows that X is the case, then he does not know that X is the case (by the contraposition of positive introspection), and the original scenario states that **p** knows that X is the case. A contradiction.

24 "Sed circa secundam propositionem videtur esse dubium, nam videtur quod possibile est quod aliquis dubitet se scire Socratem currere. Nam pono casum quod tu scias Socratem currere per tres evidentias et dubites an ille tres evidentie sufficiant ad concludendum te scire Socratem currere et volo quod dubites an requirantur quattuor que non requirantur. Quo dato arguitur quod dubites te scire Socratem currere, quia tu scis Socratem currere et consideras sufficienter an scias Socratem currere et non scis te scire Socratem currere," JEROME PARDO 1505, fol. 101vb.

25 "Et cum dicitur quod aliquis potest dubitare an tres evidentie sufficiant ad sciendum vel an requirantur plures, respondeo: Si aliquis scit aliquam conclusionem per aliquam evidentiam, ita quod assensus eius qui est scientia causatur ex assensu premissarum qui dicitur evidentia, non potest dubitare an illa evidentia sufficit ad sciendum, quia hoc esset dubitare se scire. Qui enim scit per aliquam evidentiam, scit ita esse propter ita esse sicut significatur per talem evidentiam. Ideo dubitare an illa evidentia sufficiat est dubitare an ita sit assentiendum propter illud, quod includit opinari se scire," JEROME PARDO 1505, fol. 101vb.

2.2 Gaspar Lax: *Insolubilia* (1508/1512)

Gaspar Lax discusses iterated modalities as part of the fourth question of his *Insolubilia*.²⁶ The third article of this question asks whether someone can know and fail to know the same statement,²⁷ and includes *dubia*, the third of which is whether the same sentence can be (simultaneously and in the same sense) a matter of knowledge and doubt.²⁸ In this context, Lax discusses the argument that since someone can doubt whether he knows a certain sentence (to hold), he can also have knowledge and doubts regarding that sentence (even if the meaning of the sentence remains identical).²⁹ The argument splits into two parts: the proof of the inference and the proof of the antecedent, each of which is interesting in its own right.

The first sub-proof attempts to prove the validity of the aforementioned inference:

Proof of the inference. Since assuming that, I focus on that sentence and ask whether he firmly assents to that sentence or not. If he does, let that sentence be, for instance, this one: '*Socrates is running*'. Then I argue as follows: focusing on the inference '*Socrates is running and he firmly assents to this sentence which has such meaning; therefore, he knows that sentence*', the inference is valid and known to be valid by him, and he knows the antecedent, therefore, he knows the consequent, and as a consequence does not doubt that he knows that <sentence> in that sense.³⁰

26 "Utrum aliquo casu possibili posito stet eundem simul decipi et non decipi seu errare et non errare," GASPAR LAX 1512, f2vb.

27 GASPAR LAX 1512, f6ra.

28 GASPAR LAX 1512, g4vb.

29 "[S]tat aliquem dubitare se scire aliquam propositionem in aliquo certo sensu, ergo stat aliquem scire et dubitare eandem propositionem in eodem sensu," GASPAR LAX 1512, h1va.

30 "Consequentia probatur. Quia dato illo, capiam illam propositionem et queram: vel firmiter assentiet talis illi propositioni, vel ne. Si sic, sit illa gratia exempli ista: '*Socrates currit*'. Et sic arguo: capta ista consequentia: '*Socrates currit et ille firmiter assentit illi in tali sensu, ergo ille scit illa*', consequentia ista erit bona scita ab illo et ille sciet antecedens, ergo sciet et consequens, et per consequens non dubitabit istum scire illam in tali sensu," GASPAR LAX 1512, h1va.

The proof seems to have several problematic features. First, since the segment is meant to prove that if it is possible or self-consistent to doubt one's own knowledge, it is possible to know and doubt the same thing, so it should start with the former and end with the latter. However, the argument appears to start from the assumption that someone doubts whether he knows something and ends with proving that he knows that he knows it. That could easily be fixed by explicitly citing the rule that valid inferences are possibility-preserving, but such a move is missing.

Second, the question is asked whether the agent assents firmly to the sentence, '*Socrates is running*'. One might expect that both options will be discussed but only the positive one is, which seems illegitimate unless such an assumption is included in the scenario. That said, the argument could still count as a proof that a certain (sub-)scenario is self-consistent.

Third, after proving that the agent knows that he knows something, the argument proceeds to conclude that he does not doubt that he knows that, which is superfluous. Moreover, if the last step is correct, i.e., if the existence of knowledge entails the absence of doubt, then knowledge and doubt are mutually exclusive, which goes against the intention of the argument.

The sub-proof has some interesting features. An attempt to reach the epistemic iteration '*p knows that p knows that X is the case*' is made via the principle that knowledge distributes over implication or the axiom **K**, whereas the axiom of positive introspection or the axiom **4** is not endorsed explicitly. That may be on purpose, since the second sub-proof would undermine it, rendering the entire argument inconsistent. That said, a weaker form of introspection seems to be presupposed in the proof that the agent knows that the antecedent of the proposed inference holds, i.e., that Socrates is running and that he firmly assents to the sentence '*Socrates is running*'. The first part of the

antecedent is known as a result of the agent's firm assent to the sentence and the truth of the sentence, which qualifies as knowledge in a minimal sense.³¹ However, the second part of the antecedent can only be known if some principle along the lines of 'if p firmly assents to ϕ , p knows that p firmly assents to ϕ ' is presupposed, and that constitutes some form of introspection.

The second sub-proof introduces a scenario in which Socrates, based on observational data, firmly believes that the king is asleep, but doubts whether such evidence is sufficient.³² As a tacit assumption, the king is asleep, whence Socrates does actually know that the king is asleep as a result of having a firm, evidence-based and factually correct belief. In this scenario, Socrates doubts that he knows that the king is asleep; let us consider the inference: '*Socrates firmly assents to this sentence precisely for those reasons and Socrates knows the sentence (to hold), therefore those reasons sufficiently establish the knowledge of the sentence in question*'. The inference is held to be valid and known to be valid by Socrates, but its consequent is, by assumption, not firmly assented to by him, which means that he cannot firmly assent to the antecedent. However, Socrates is held to firmly assent to the first part of the antecedent. Therefore, Socrates does not firmly assent to knowing that the king is asleep, whence he does not know that he knows that, which implies that he doubts that he knows that the king is asleep.³³

31 For Lax's analysis of 'scire' and 'scire propositionem' see GASPARELLO LAX 1512, g1rb-g2ra.

32 "Iam probatur antecedens confirmationis: volo quod Socrates propter aliquas apparentias firmiter assentiat huic propositioni, scilicet '*rex dormit*', dubitet tamen an ille cause sint satis sufficientes ad hoc ut illa propositio sciatur in illo sensu et non habeat alias causas nec apparentias," GASPARELLO LAX 1512, h1va.

33 "Hoc dato sic arguo: ille dubitabit se scire illam propositionem in illo sensu, ergo propositum. Antecedens probo. Capió istam consequentiam: '*ille assentit firmiter solum propter illas causas illi propositioni et ille scit illam, ergo ille sunt sufficientes ad hoc quod talis propositio sciatur in tali sensu*'. Consequentia ista erit bona, scita a tali. Et ipse non firmiter assentiet consequenti, ut patet ex casu, ergo non firmiter assentiet antecedenti. Et firmiter assentit prime parti, ergo non firmiter assentiet secunde. Et per consequens non sciet se scire illam nec dissentiet tali propositioni, ergo dubitabit se scire illam," GASPARELLO LAX

Lax replies to this argument in two theses. The first is that someone can doubt whether he knows a certain non-self-referential sentence (to hold), proved by positing the scenario in which the agent is in doubt regarding his physical location, holding it possible that he is in Rome by a miracle, in which case he doubts whether, while being in Rome, he knows that the pope is asleep.³⁴ Such rather artificial scenarios aside, the second thesis states that the question of whether someone can be in doubt regarding his own knowledge is undecidable and that alternative plausible solutions to the sophism can be formulated.³⁵

2.3 Pardo, Lax and the Brito-Italian Tradition

Lax's and Pardo's texts have several interesting features. First, the criticism of the axiom of positive introspection takes two forms. The first emphasises the independence of first-order and second-order knowledge; a similar argument was introduced by John Wyclif and Paul of Venice.³⁶ The other employs the

1512, h1va. Note that the final step requires that the absence of knowledge implies doubt.

34 "Prima. Stat bene aliquem dubitare se scire aliquam certam propositionem non reflexivam in aliquo certo sensu. Propositio probatur, quia stat bene me nullo facto miraculo dubitare an sim Rome, dubitando an Deus fecerit aliquod miraculum, et ex consequenti me dubitare an sciam Rome illam: '*Papa dormit*' in illo sensu," GASPAR LAX 1512, h1vb.

35 "Secunda propositio. Sustenable est quod aliquis possit dubitare se scire aliquam certam propositionem non reflexivam in aliquo certo sensu non reflexivo, sciendo adhuc ipsum non esse nisi in illo loco. Et oppositum huius est sustenable. Nec potest evidenter aliquod istorum vel eius oppositum concludi. Et dicendo primo modo, diceretur ad illud quod tangis quod non staret illud te habente illos discursus, et proportionabiliter diceretur in aliis argumentis que possent contra hoc applicari. Dicendo secundo modo facile solveretur argumentum quod tangis dicendo quod non est possibile quod propter tales causas precise assentias tali et scias illam et dubites an ille sint sufficientes ad hoc quod scias illam," GASPAR LAX 1512, h1vb. Lax pays some further attention to scenarios involving self-referential phenomena and ultimately notes that a similar reasoning applies to other forms of iterated modalities, such as the problem of whether someone can doubt that he is in doubt (see GASPAR LAX 1512, h2ra).

36 See JOHN WYCLIF 1893, 184 (as Mark Thakkar kindly informed me, Dziewicki's edition of this passage is based on a particularly unreliable manuscript), analysed in HANKE

notion of evidence which allows that someone has a factually correct and evidence-based belief while being uncertain about the sufficiency of such evidence; similar arguments were employed by Peter of Mantua and Cajetan of Thiene³⁷ and Pardo's own reply echoes the approach of Cajetan of Thiene.³⁸ Second, the criticism of the axiom of positive introspection is introduced as part of proving the consistency of the scenario in which an agent doubts that he knows something. That is insufficient, but precisely what a scholastic author, knowledgeable upon epistemic logic, might be expected to do since it amounts to attacking a widely accepted strategy. Third, recall that when Pardo discusses the same scenario, he argues *only* against one particular threat to its consistency, arguably against the one which he views as the most reputable, which too can be interpreted as a historical contingency. These observations document that while Pardo and Lax came up with interesting innovations, they were to a significant degree perpetuators of the Brito-Italian logical tradition.

3. Epistemic Closure in *Posterior Analytics* Commentaries

In modern epistemic logic, the most elementary axiom, in the sense that the hierarchy of axiomatic systems is built up by adding further principles to it, is the principle that knowledge distributes over implication or the axiom **K**: if **p** knows that A implies B and **p** knows that A, then **p** knows that B.³⁹ In late-medieval logic, this principle plays multiple roles in various contexts: it is an

2018(1), 169–173, and PAUL OF VENICE 1499, fol. 81vb, analysed in HANKE 2018(2), 231–233.

37 See PETER OF MANTUA 1492, f7va (analysed in HANKE 2018(1), 174) and CAJETAN OF THIENE 1494, fol. 17ra (analysed in HANKE 2018(2), 234).

38 See CAJETAN OF THIENE 1494, fol. 17rb, analysed in HANKE, 2018(2), 241 (the same paper analyses the interesting contribution of Paul of Pergula).

39 See HUGHES, CRESSWELL 1996, 359–368. The same source can be consulted for the terminology of modern modal logic used throughout this paper.

inference rule in *consequentia* treatises (Ralph Strode), challenged in *insolubilia* treatises (Paul of Venice), and is used in setting up and solving sophisms in *de scire* treatises (William Heytesbury).⁴⁰ Here, a widely popular sophism whose precursors were introduced by Peter of Mantua and Paul of Venice will be discussed.⁴¹ The two Italians proposed similar sophisms targeting the notion of appearance, whose influence was acknowledged by the sixteenth-century authors. Peter of Mantua discussed the scenario in which two bodies are equidistant from an observer, to whom they appear equally long, namely one foot. If one of the bodies begins moving farther away from the observer to the point where it appears to be merely half a foot long, it would simultaneously appear to be half a foot long (by direct observation) and a foot long (by the earlier observation that the two bodies are equal).⁴² To outline Peter's solution, he makes two notes, each of which would solve the problem. First, he allows that the same object appears to be both a foot long and half a foot long based on different observational data (*secundum diversas apparentias*). Second, he introduces restrictions on the inferences containing the verb 'to appear' which invalidate the argument.⁴³

40 These examples are introduced in the aforementioned publications of Boh and Hanke.

41 The fact that the sophism is discussed by Peter of Mantua and Paul of Venice suggests that there could have been earlier British proponents of this sophism. While these were not mentioned in John Mair's circle, that might have been due simply to practical availability.

42 "Quarto. (...) per idem habetur in casu quod idem appareret tibi pedale et semipedale (...) Ponendo quod **a** et **b** pedalia distent equaliter a te que tibi appareant pedalia et equalia. Deinde incipiat removeri **b** per magnam distantiam et removeatur quousque apparebit tibi solum semipedale te bene sciente quod est pedale. Et sit iam medium instans hore in quo **b** appareat semipedale et arguitur quod **b** apparet tibi pedale et quod apparet semipedale. Quod enim **b** appareat tibi semipedale, apparet ex casu: **b** enim apparet sub duplo minori angulo quam ante apparebat et cetera sunt paria, igitur **b** apparet tibi minus quam ante apparebat. Sed quod **b** appareat tibi pedale arguitur, quia tu scis **b** esse pedale, quia scis **a** et **b** esse equalia, igitur credis **b** esse pedale. Et qualitercumque credis, taliter apparet tibi esse, igitur apparet tibi **b** esse pedale," PETER OF MANTUA 1492, g1vb.

43 "Ad quartum dicitur quod stat idem apparere pedale et semipedale secundum diversas apparentias. Negatur tamen in casu illo quod **b** appareat minus quam pedale. Et non

Paul of Venice's formulation of the problem is modified but similar; the sophism is part of the *de sensu composito et diviso* treatise of *Logica magna*.⁴⁴ His scenario assumes that there are three bodies named 'a', 'b' and 'c', such that **a** is imperceptibly longer than **b** and **b** is imperceptibly longer than **c**, but **a** is visibly longer than **c**. In that scenario, **a** and **b** appear to be equal, as do **b** and **c**, but **a** does not appear to be equal to **c**. However, given the apparent equality of **a** with **b** and of **b** with **c** and the transitivity of equality, **a** must appear to be equal to **c**: the inference '*a is equal to b and b is equal to c, therefore a is equal to c*' is known to be valid, therefore it appears to be valid, and the antecedent appears to hold, therefore, the consequent must appear to hold as well.⁴⁵ The argument proceeds in an unexpected way: rather than suggesting that apparent truth is preserved by inferences which are known to be valid, it insists that apparent truth (or validity) distributes over implication, which seems to be more controversial. Still, Paul's reply focuses on a different aspect of the argument and denies that the antecedent is apparently true, even though its parts are.⁴⁶

valet hoc argumentum: '*c, d et b apparent equalia, sed c apparet minus quam pedale, igitur b apparet minus quam pedale*', quia posito quod **c** esset unum minus quam pedale quod esset inter **a** et **b** in tanta distantia quod appareat semipedale, antecedens est verum et consequens falsum," PETER OF MANTUA 1492, g2ra.

44 *Logica magna* will be treated here as Paul of Venice's authentic work, such as it was viewed in John Mair's circle. That said, nothing important in this study rests on that assumption and the problem will not be discussed in detail. For the most recent discussion of this problem, see PAUL OF VENICE forth.

45 "Iuxta dicta solet dubitari persuadendo aliqua duo eidem apparere equalia et inequalia sic: ponatur a remotis **a, b, c**, et sit **a** insensibiliter maius **b**, sic quod appareat tibi **a** et **b** esse equalia, sit etiam **b** insensibiliter maius **c**, ita quod **b** et **c** appareant etiam tibi equalia, sed sit excessus **a** super **c** sensibilis, sic quod bene sentias **a** esse maius **c**. Isto posito patet quod **a** non apparet tibi equale **c**. Sed probatur quod sic. Et facio istam consequentiam: '*hoc a est equale huic b et hoc b est equale huic c, igitur hoc a est equale huic c*'. Ista consequentia apparet tibi bona, quia bene scis quod illa est bona, et antecedens apparet tibi verum, ergo et consequens," PAUL OF VENICE 1499, fol. 77va. In the only currently known manuscript of *Logica magna* (PAUL OF VENICE (ms.)), the corresponding passage is *Città del Vaticano*, Biblioteca Apostolica Vaticana, Vat.lat. 2132, fols. 98vb-99ra, but given its rarity, it is unlikely to have been the actual source.

46 "Ad primum respondetur negando quod antecedens illius consequentie apparet mihi

In John Mair's circle, Paul's sophism became a commonly discussed problem related to the axiom **K** and was referenced as having been introduced by Paul of Venice, with two notable adaptations. First, the sixteenth-century formulations replace 'appearance' with 'knowledge' and 'assent', while retaining the scenario and agreeing that equality is transitive or Euclidean and viewed as such. Second, while Paul of Venice was attributed with the reformulated version of the sophism, the references were not entirely precise. For instance, Antonio Coronel, who confirms the popularity of the sophism by labelling it as '*commune*', claimed that the problem was contained in the *de scire et dubitare* chapter of *Logica magna*.⁴⁷ Later on, Domingo de Soto would attribute the sophism to '*novi posterioristici*', probably referring to the contemporary Parisian logician, on top of mentioning Paul of Venice's '*sophisms de scire et dubitare*'.⁴⁸

2.1 John Mair: *In Petri Hyspani Summulas Commentaria* (1503/1505)

In addition to the chapters corresponding to Peter of Spain's treatises, John Mair's commentary on Peter of Spain's *Summulae* incorporated additional material, including *Liber Posteriorum*.⁴⁹ The adaptation of Paul of Venice's scenario is discussed in the first chapter of *Liber posteriorum*, which discusses the Aristotelian thesis that all knowledge and every doctrine comes about from pre-existing knowledge. To account for the difficult aspects of the theory,

verum, quia licet utraque pars illius antecedentis apparet mihi vera, non tamen apparet mihi quod utraque pars istius sit vera," PAUL OF VENICE 1499, fol. 77va.

47 See below for the details.

48 "Per haec aperitur via respondendi ad confirmationem, quae ex sophismatibus Pauli Veneti de scire et dubitare solent huc afferre novi posterioristici...", DOMINGO DE SOTO 1554, fol. 81rb). The passage is discussed in HANKE forth.(2), where the relation to John Mair's circle was not recognised.

49 Note that there is also an earlier 1503 separate edition of *Liber Posteriorum*, see JOHN MAIR 1503.

Mair discusses a series of *dubia*, the third of which addresses the thesis that the conclusion of a syllogism is cognised as soon as its premises are.⁵⁰ As that formulation appears too strong when applied to a human agent, Mair offers the following reformulation: as soon as an agent assents to both premises of a syllogism presented to him in their proper form, he assents to the conclusion, provided that he knows that the inference in question is valid and that all valid inferences which do not include any form of self-reference are truth-preserving; the same idea is, with a slight adaptation, restated in terms of knowledge rather than assent.⁵¹ There are two points of note. First, the rule to be challenged is formulated in terms of assent and knowledge, rather than apparent truth. Second, the complexity of the principle is reminiscent of similar approaches in the earlier scholastic tradition; note, in particular, the mentioning of self-referential phenomena, significant to John Mair's circle as an influence arising from Roger Swyneshed's suggestion that validity does not entail truth-preservation.⁵² The plurality of formulations bridges fourteenth- and fifteenth-century epistemic logic and sixteenth-century epistemology by giving two alternative expressions of what appears to be the intuitive core of

50 "Dubitatur de veritate huius propositionis: '*cognitis maiore et minore simul tempore cognoscitur conclusio*'," JOHN MAIR 1505, et5va.

51 "Ad tertium dubium, quod sic intelligitur quod quis potest maiori assentire antequam conclusioni assentiat et pari forma minori antequam conclusioni assentiat, sed quamprimum illis in forma positis assentit et scit consequentiam esse bonam, mox conclusioni assentit, dummodo sciat quod ex vero non sequitur propositio falsa non reflexiva," JOHN MAIR 1505, et6rb. This formulation is held equivalent to the following: "Si aliqua consequentia est bona, scita esse bona ab aliquo, si sciatur ita esse sicut significatur per antecedens, scitur ita esse sicut significatur per consequens, dummodo non repugnat consequenti sciri," JOHN MAIR 1505, et6rb-va. The second formulation is closer to the formulation typical of medieval treatises on consequences.

52 There is a growing body of research on this tradition. The fundamental editorial work includes ROURE 1962, SPADE 1979 and BRICOT 1986 and PAUL OF VENICE forth., and a number of recent and forthcoming publications by Stephen Read, including READ 2020(1) and READ 2020(2). For the general context of these approaches, see SPADE, READ 2018, for the philosophical context, see DUTILH NOVAES 2008. For John Mair's circle, see ASHWORTH 1974(2), 112-113 and ASHWORTH 1977; D'ORS 1986; HANKE 2012, 2013 and 2014. For the role of paradoxes in scholastic epistemic logic, see (among others) HANKE forth.(1).

the axiom **K**.

As one of the counter-examples to the principle, Mair restates Paul of Venice's scenario: let us posit that there are three bodies possessing the physical properties described by Paul of Venice. The scenario is held to be possible (i.e., acceptable for the sake of argument). Now let us assume that the following inference is proposed to Socrates: '*things that are equal to the same thing are also equal to one another, a and c are equal to the same thing (namely to b); therefore, a and c are mutually equal*'. In this scenario, Socrates is assumed to know that the inference is an instance of DARII, i.e., a valid syllogistic inference. The first of the premises is a 'common notion' (*communis animi conceptio*), whence Socrates must assent to it and the second premise is assented to by hypothesis. However, Socrates would not assent to the consequent (based on perceptual evidence). As a result, the scenario is a counter-example to the axiom **K** (in one form or another).⁵³

Mair provides a two-step reply. First, he considers the argument irrelevant (to the context of *Posterior Analytics*), since the second premise is not cognised in a sense relevant to the Aristotelian notion of scientific knowledge or demonstration. Second, he denies that Socrates would assent to both premises in this scenario: if Socrates knew that the consequent of the aforementioned inference is false (based on perceptual evidence), that validity entails truth-preservation (in all relevant cases) and that the first premise is true, he would *immediately* dissent to the second premise, and his intellect would re-e-

⁵³ "Contra regulam (...) instatur. Bene sequitur: '*quecumque sunt equalia uni tertio sunt equalia inter se, a et c sunt equalia uni tertio (scilicet b), ergo a et c sunt equalia*'. Stat quod aliquis assentiat antecedenti et tamen non assentiat consequenti, esto quod sciat consequentiam esse bonam, igitur minor patet. Posito quod **a** et **b** sint duo corpora ferme equalia, sed **b** **a** exuperet imperceptibiliter et **c** **b** excedat etiam imperceptibiliter, sed **c** **a** sensim excedat. Totus casus est possibilis, maior est communis animi conceptio, ergo assentit illi Socrates, et assentit minori per ypotesim, igitur toti antecedenti et non assentit consequenti, et pono quod sciat consequentiam esse in DARII," JOHN MAIR 1505, et6va.

value the perceptual evidence as flawed.⁵⁴

2.2 Antonio Coronel: *Commentaria in Posteriora Aristotelis* (1510/1528)

Antonio Coronel came up with what is fundamentally the same scenario to challenge the same principle, but developed it in a different way. The agent in the scenario is assumed to have assented to the premises of: “*things that are equal to the same thing are also equal to each other, a and c are equal to the same thing (namely to b); therefore, a and c are equal to each other,*” after which he would form that inference. In that case, the agent would assent to both premises and the inference evident to him, but not to the conclusion. The reasons are the posited visual counter-evidence and the fact that nobody can both assent to and dissent from the same statement at the same time.⁵⁵

Coronel presents three possible solutions. The first two are attributed to Paul of Venice and Cajetan of Thiene, the unattributed third is refined into

54 “Respondetur: quicquid sit argumentum, non est contra mentem Aristotelis, quia talis non cognoscit minorem, capiendo cognoscere sicut Aristoteles capit. Sed ad argumentum in se, quia is non assentit antecedenti totali, immo dissentit minori, postquam videt consequens esse falsum, et ad sensum hoc percipit, et scit quod ex vero non sequitur falsum (saltem extra reflexivas) et scit maiorem esse veram, statim minori per intellectum dissentit, quia sensum delirantem ille castigat,” JOHN MAIR 1505, et6va.

55 “Ad secundum dubium in quo queritur utrum cognita maiore et minore simul tempore cognoscatur conclusio. (...) Primo arguitur in aliquo casu stat assentire antecedenti et consequentiae et non conclusioni, igitur conclusio falsa. Antecedens probatur. Ponatur casus communis: sint tria corpora coram te **a**, **b**, **c**, **a** insensibiliter maius **b**, taliter quod per sensum non possis iudicare **a** esse maius **b**, sed precise quod est ei equale, et **b** insensibiliter maius **c** propter eandem causam, sed **a** sit sensibiliter maius **c**, taliter quod sensu percipere potes **a** esse maius **c**. Volo quod per totam horam preteritam tu assentiebas isti copulative: ‘*quecumque sunt equalia uni tertio sunt equalia inter se, a et c sunt equalia uni tertio, scilicet b*’. Et in hoc instanti primo formes completam consequentiam, sic dicendo: ‘*ergo a et c sunt equalia inter se*’. Et arguitur sic: In hoc instanti habes assensum maioris et minoris et consequentiae (cum sit tibi evidens). Et non assentis conclusioni, quod patet: dissentis ei, ergo non assentis ei, cum non possis eidem propositioni simul et semel assentire et dissentire. Antecedens probatur, nam sensu percipis **a** esse maius **c**, ergo propositum. Hoc argumentum est commune in hac materia,” CORONEL 1510, fol. 4rb–va.

Coronel's own solution. Needless to say, the correspondence between the actual sources and Coronel's presentation is rather loose. Since this is common to other authors from John Mair's circle, there appears to be a common source to this (mis)conception.

First, Paul of Venice is claimed to have admitted the scenario as possible while denying that the agent assents to the antecedent of the aforementioned inference, even though he assents to each of its parts. The assent to the entire antecedent is held to be different from the two partial assents. This position is referenced back to Paul's *Logica, capitulo de scire et dubitare*,⁵⁶ which is incorrect for several reasons. First, the scenario is discussed in a chapter other than that indicated. Second, Coronel shifts from apparent truth to assent, which changes the principles at stake. Third, the theory of mental acts seems imposed on Paul's text.

Coronel rejects the solution by suggesting that such an agent would surely agree that conjunction introduction is legitimate, which should guarantee the existence of the required act of assent. In other words, assent can be assumed to be closed under conjunction introduction, which Coronel supports with two reasons. First, the opposite assumption postulates an agent with an unrealistic degree of logical incompetence and, as Longeway noted, ultimately results in an infinite regress.⁵⁷ Second, as an interesting move from pure logic to psychology, the assent to the conjunctions is assumed to be generated *causally* by the pre-existing assents to the sub-formulas⁵⁸.

56 "Ad hoc respondet Paulus in sua *Logica*, capitulo de scire et dubitare casu admisso, negando quod assentias antecedenti, licet assentias maiori et minori. Assensus totius antecedentis est distinctus ab assensu partium et tu non habes illum tertium assensum," CORONEL 1510, fol. 4va.

57 LONGEWAY 2009, 401–402.

58 "Hec solutio nulla est, nam bene sequitur: quelibet pars huius copulative est vera, ergo tota copulativa est vera. Sed iste stante toto casu, si formaret istam consequentiam, assentiret ei et antecedenti eius, ergo assentiret consequenti. Ad hoc argumentum respondet ipse, quod ille talis non assentiret consequentie. Sed hoc nichil est, nam suppono

The second strategy is attributed to Cajetan of Thiene's commentary on Heytesbury's *De scire et dubitare*. According to Coronel's formulation, the assents to different parts of the inference are performed by different cognitive powers: intellect assents to the underlying mathematical axiom while senses assent to two bodies being equal to a third body. For that reason, someone can assent to both premises and to the inference but not to the conclusion.⁵⁹ It is not clear to which passage Coronel is referring: while certain parts of Cajetan's commentary on Heytesbury use the distinction between perceptual and intellectual, that appears to be limited to different modes of reference to an object within specific scenarios.⁶⁰

Coronel's reply is distorted in printed editions of the text, but its content is relatively comprehensible: the distinction does not solve the problem, since intellect follows senses in the absence of a counter-argument, and the entire argument could be restated for an angel or a soul separated from the body. Coronel does not elaborate on the second point and it is not clear how such reformulations should work, when the argument relies on *perceptual* distinguishability.⁶¹

The third position naturalises the issue and the discussion of the axiom **K** turns into a debate on whether cognitive changes are instantaneous or suc-

quod sciat aliquid in logica et ex consequenti quod sciat ad veritatem copulative sufficere cuiuslibet partis veritas. (...) Tum tertio: ille tertius assensus, si ponendus est, causatur ab assensibus partium copulative. Sed iste per totam horam preteritam habuit assensum partium copulative, ergo habuit illum assensum," CORONEL 1510, fol. 4va.

59 "Ideo ponitur alia solutio, que est Gaethani de Thienis in *Commento de scire et dubitare Hentisberi*, quod in illo casu Socrates assentit antecedenti, sed non per eandem potentiam: assentit maiori per intellectum et non per sensum, minori opposito modo. Unde non est inconveniens assentire maiori et minori et consequentie et non conclusioni, quando non assentitur maiori et minori per eandem potentiam," CORONEL 1510, fol. 4va.

60 See CAJETAN OF THIENE 1494, fols. 19ra and 19va.

61 "Hoc solutio nulla est, nam cum primo sensus assentit minori, intellectus ei assentit, cum non habeat rationem in oppositum. Tum tertio, quia argumentum potest fieri de uno angelo vel de anima separata," CORONEL 1510, fol. 4va. Cf. CORONEL 1528, fol. 5va.

cessive, how to analyse beginning and ceasing, the nature of the causal potency of certain cognitive acts, and so on, starting with the thesis that as soon as Socrates assents to the inference and dissents from the conclusion, the assent to the second premise ceases to exist.⁶² Such naturalisation of logic is coherent with other issues discussed by Coronel, such as quantitative limitations upon the capacity of the human mind (which could be a sign of continuity with the Oxford Calculators).⁶³

2.3 Juan de Celaya: *Expositio in libros Posteriorum Aristotelis* (1517/1521)

Juan de Celaya addresses Paul of Venice's scenario while discussing the validity of the axiom **K** applied to syllogistic inferences.⁶⁴ The scenario is formulated for Socrates playing the role of the agent.⁶⁵ The most significant difference compared with Mair and Coronel is that the inference proposed to Socrates is simply '*a is equal to b and b is equal to c; therefore, a and c are mutually equal*'. In such a scenario, Socrates is held to assent to both premises based on empirical evidence and to the proposed (syllogistic) inference, since he is a

62 The series of propositions and counter-arguments starts with the following statement: "Ideo ponitur tertia solutio que talis est: in primo instanti in quo Socrates assentit consequentie et dissentit conclusioni per primum non esse corrumpitur assensus minoris. Tunc dicitur ad formam argumenti cum sic arguitur: Socrates in hoc instanti assentit maiori et minori etc., nego istam, quia hoc instans est primum non esse assensus minoris," CORONEL 1510, fol. 4va. For the details of Coronel's position, see LONGEWAY 2009, 403–406.

63 See LONGEWAY 2009, 406–418. For a similar discussion in Soto's commentary on *Posterior Analytics*, see HANKE forth.(2).

64 "Queritur circa hunc textum an illa secunda conclusio Philosophi, scilicet cognitis maiore et minore simul tempore cognoscitur conclusio sit vera. (...) Quarta conclusio est ista: impossibile est cognoscere maiorem et minorem et bonitatem consequentie ad sensum declaratum quin in eodem instanti cognoscatur conclusio," JUAN DE CELAYA 1517, fols. 16vb and 17ra. There is a second edition, issues 1521, see JUAN DE CELAYA 1521.

65 "Secundo principaliter arguitur: possibile est Socratem assentire maiori et minori et bonitati consequentie non assentiendo conclusioni, igitur illa conclusio falsa. Antecedens probatur: volo quod sint tria corpora ante Socratem, scilicet **a**, **b** et **c**, **a** sit imperceptibiliter maius **b**, similiter **b** sit imperceptibiliter maius **c**, **a** vero sit perceptibiliter maius **c**," JUAN DE CELAYA 1517, fol. 17va.

competent logician – a point repeated several times.⁶⁶

Celaya presents three solutions to the problem, the first two of which are attributed to Paul of Venice and Cajetan of Thiene. Celaya's presentation of Paul of Venice's solution to the problem does not differ from Coronel's in any significant way, except for the fact that the reference is correct.⁶⁷ The suggestion that Socrates would assent to each premise separately but not to their conjunction is dismissed by pointing out its inconsistency with Aristotelian epistemology and by insisting that Socrates is assumed to be a competent logician who is paying attention to the problem.⁶⁸ Overall, this seems to be Coronel's reply, dressed up in different clothes. Similarly, Celaya's presentation and criticism of Cajetan's position is identical to Coronel's (minus the

66 "Et proponatur Socrati iste sillogismus: '*a est equale b et b est equale c, ergo a et c sint equalia*'. Tunc Socrates assentiet maiori et minori et bonitati consequentie et non assentiet conclusioni, quia habet scientiam de eius contradictorio, igitur antecedens verum. Quod Socrates assentiet maiori et minori patet, quia ad experientiam apparent sibi vere maior et minor, cum non possit distinguere seu discernere excessum **a** corporis supra **b** corpus nec excessum **b** corporis supra **c** corpus. Quod Socrates assentiet bonitati consequentie probatur: suppono quod illi tres termini **a**, **b**, **c** sint termini discreti significantes illa tria corpora. Socrates est bonus logicus et advertit circa bonitatem illius consequentie, ut suppono, et illa consequentia est bonus sillogismus expository, ergo assentit illi," JUAN DE CELAYA 1517, fol. 17va.

67 "Ad hoc argumentum respondet Paulus Venetus in prima parte *Logice magne*, capite de *sensu composito et diviso*, concedendo antecedens et negando consequentiam. Ratio assignatur ab eo, quia licet Socrates assentiat maiori et minori, non tamen assentit toti antecedenti. Nam non habet Socrates in illo casu unum assensum circa totum antecedens, sed duos," JUAN DE CELAYA 1517, fol. 17vb.

68 "Hec solutio parum aut nihil valet, nam obviat Philosopho, qui oppositum asserit. Item si Socrates est bonus logicus, considerat circa copulativam, que est antecedens, et assentit cuilibet parti copulative, ergo assentiet toti copulative, cum sciat bene ad veritatem copulative sufficere utramque partem principalem esse veram. Et per consequens habebitur quod Socrates assentit toti antecedenti et bonitati consequentie et non conclusioni, quod est contra Philosophum et ipsummet Paulum Venetum," JUAN DE CELAYA 1517, fols. 17vb. The emphasis on attention may be significant: the argument assumes that Socrates agrees to two statements and pays attention to whether their conjunction holds and is familiar with the rule of conjunction introductions. In general, this amounts to the axiom **K** extended by the requirement of attention, and such extensions are characteristic of Italian scholasticism (see HANKE forth.(1)).

flaws).⁶⁹ These similarities suggest that the two commentaries are not independent.

The third solution develops the scenario to account for possible objections. As the first step, Celaya states that Socrates could assent to both premises without assenting to the inference '*a is equal to b and b is equal to c; therefore, a and c are mutually equal*'.⁷⁰ Second, if the scenario is modified by additionally positing that Socrates contemplates whether the inference is valid, it follows that he would assent to its validity, which would immediately prevent him from assenting to both premises as a result of his logical competence.⁷¹ The strategy can be restated even if one assumes that belief-changes such as ceasing to assent are successive processes that do not permit an instantaneous change.⁷²

69 "Ideo aliter respondet Gaietanus de Tyennis, commentator Hentisberi, concedendo quod in illo casu Socrates assentit maiori et minori et bonitati consequentie et etiam toti antecedenti et non conclusioni, sed non per eandem potentiam, sed per diversas. Nam per potentiam sensitivam assentit antecedenti et per potentiam intellectivam assentit bonitati consequentie. Nec hoc est contra Philosophum, quia Philosophus intelligit per unam et eandem potentiam. Ista solutio etiam non valet. Nam quamprimum sensus assentit alicui propositioni, intellectus assentit eidem, nisi habeat aliquam rationem fortem in oppositum. Insuper argumentum potest deduci de anima separata vel de angelo," JUAN DE CELAYA 1517, fol. 17vb.

70 "Propter hoc igitur aliter est respondendum ad argumentum. Unde ad formam argumenti negatur antecedens et ad probationem admissio casu, concedo quod assentiat maiori et minori, nego tamen quod assentit bonitati consequentie," JUAN DE CELAYA 1517, fol. 17vb.

71 "Et ad probationem, si ponas quod advertat circa bonitatem consequentie, concedo quod assentiet bonitati consequentie, nego tamen quod tunc assentiet maiori et minori, immo quamprimum assentit bonitati consequentie, dissentiet alicui premissarum, postquam est bonus logicus. Nam videbit clarissime bonitatem consequentie et falsitatem consequentis, eliciet ergo statim falsitatem antecedentis. Et per consequens necessario dissentiet tunc alicui premissarum, et sic numquam habebuntur illi tres assensus sine assensu conclusionis," JUAN DE CELAYA 1517, fol. 17vb.

72 "Et si supponas quod quilibet assensus debeat corrumpi successive, facile est etiam respondere, nam numquam habebitur in illo casu assensus bonitatis consequentie, donec aliquis assensus premissarum fuerit destructus. Et si dicas: quid impedit Socratem habere assensum bonitatis consequentie, postquam est bonus logicus et advertit circa illam, ad hoc dico quod dissensus conclusionis impedit, nam Socrates ideo quia est bonus logicus videt falsitatem consequentis et credit antecedens esse verum, ea de re dubitat

Third, Celaya introduces the following objection. Assume that Socrates, who is a competent logician, regards a certain conclusion as conjectural, and then a proof of the conclusion is presented to him, which he contemplates. Afterwards, Socrates starts contemplating the syllogistic premises of the proof that are assumed to have the same degree of apparency (*apparentia*). Based upon these assumptions, Socrates would have to assent to both premises: he must assent to one of them, but he cannot favour either of them, since they are equally evident. He is assumed to have assented to the inference. However, he would not assent to the conclusion (since knowledge and conjecture are mutually exclusive), which is regarded as undesirable or simply false.⁷³

Celaya denies that Socrates would assent to both premises in this scenario and offers two different strategies without indicating a clear preference for either. The first turns the original argument around: the parity of evidence may be what prevents Socrates' assent to either premise. The second suggests that Socrates could assent to one of the premises simply as a matter of a deliberate (rather than rationally warranted) decision.⁷⁴

de bonitate illius consequentie," JUAN DE CELAYA 1517, fol. 17vb.

73 "Contra istam solutionem arguitur. Sequeretur quod si Socrates haberet opinionem alicuius conclusionis et adducatur illi demonstratio illam demonstrans et advertat Socrates immediate circa bonitatem consequentie et sit bonus logicus et postea advertat circa maiorem et minorem simul et semel et habeat tantam apparentiam adequate erga unam premissam sicut erga aliam, tunc Socrates assentiet bonitati consequentie et maiori et minori et non assentiet conclusioni. Consequens est falsum, igitur. Quod assentiet bonitati consequentie non est dubium. Quod assentiat maiori et minori probatur: alicui illarum assentiet postquam habet motivum. Et non est maior ratio de una quam de alia, postquam habet equalem apparentiam erga ambas, ergo ambabus assentiet. Quod non assentiet conclusioni probatur: dato opposito sequitur quod habebit opinionem et scientiam de eadem conclusione, quod est impossibile, igitur nullo modo assentiet conclusioni," JUAN DE CELAYA 1517, fols. 17vb-18ra.

74 "Ad hanc replicam respondetur negando sequelam pro illa parte, scilicet quod assentiet maiori et minori. Et ad probationem: Potest negari quod assentiet alicui illarum premissarum, postquam ex casu habet tantam apparentiam erga unam sicut erga alteram. Potest etiam concedi quod alicui illarum assentiet. Et negatur quod non sit maior ratio de una quam de alia. Et pro ratione debet assignari libertas voluntatis. Nam postquam ha-

4. Bradwardinian Semantics and Epistemic Sophisms

As part of developing two epistemic scenarios in *Regule solvendi sophismata*, Heytesbury confronts the Bradwardinian idea that sentential meaning is closed under entailment.⁷⁵ Since he treats knowledge as fundamentally linguistic by typically using phrases such as ‘*p* knows the sentence ξ ’, the idea that sentential meaning is closed under entailment has significant implications for his logic.⁷⁶ In John Mair’s circle, that problem appears to be relatively rare and only one representative thereof can be introduced at this point. This could obviously mean that some sources have been omitted and will resurface in future. However, there is an argument to be made why such debate could be genuinely rare in John Mair’s circle. The idea that sentential meaning is closed under entailment was for Heytesbury tied to the treatment of semantic paradoxes. As opposed to the circle of the Oxford Calculators, where that idea was coined by Bradwardine and Heytesbury, Mair’s students were more likely to develop the treatment of paradoxes introduced by Roger Swyneshed, which implies a critical stance towards Bradwardinian seman-

bet equalem apparentiam circa illas, voluntas pro libito suo potest se determinare ad partem quam voluerit, utputa ad maiorem vel minorem,” JUAN DE CELAYA 1517, fols. 18ra. Through the ‘*non est maior ratio*’ part, this passage is tied to the analysis of the no-no paradox; for a philosophical analysis of the problem based on the scholastic material, see READ 2006, for Mair’s discussion of the paradox, see HANKE 2012, 167–168.

⁷⁵ See THOMAS BRADWARDINE 2010, which replaces the earlier edition in ROURE 1970. Furthermore, an interesting debate of Bradwardinian semantics is published in RAHMAN, TULENHEIMO, GENOT 2008. For Heytesbury’s discussion as related to epistemic sophisms, see HANKE 2021 and HANKE forth.(1), where further references are discussed (together with the positions of Heytesbury and Paul of Venice). The view that sentential meaning is closed under entailment develops the treatment of semantic paradoxes discussed in the first chapter of *Regule solvendi sophismata* (which is, together with other sources, discussed in PIRONET 2008). As suggested by Yrjönsuuri, Heytesbury’s position can be explained by the influence of Thomas Bradwardine: see YRJÖNSUURI 2008, 599–600.

⁷⁶ The idea that linguistic objects, specifically sentences, are the proper subject-matter of knowledge was in the contemporary context famously introduced by William of Ockham (see SPADE, PANACCIO 2019 referencing further sources). To what extent Heytesbury is developing the same notion of knowledge is not clear: his choice might have been more directly influenced by the genre of *obligationes* in which the problem is presented.

tics.⁷⁷ The one currently known exception is Mair’s predecessor Jerome Pardo who introduced the problem as part of the seventh counterexample to the principle that the same proposition cannot be known and conjectured at the same time. The sophism he discusses can be traced back to Heytesbury and Paul of Venice. To show his probable sources, let us consider four formulations of the problem offered by Heytesbury in *Regule solvendi sophismata*, by Paul of Venice in his *Logica magna* and *Sophismata*, and by Pardo in his *Medulla dialectice*.⁷⁸ The scenario consists in positing that an agent knows that someone is either Socrates or Plato, while doubting or not knowing which. The comparative analysis suggests that there are two different formulations of the problem, the first proposed by Heytesbury and by Paul of Venice in *Sophismata*, the other by Paul of Venice in *Logica magna* and by Pardo:

<i>Regule solvendi sophimata</i> ⁷⁹	<i>Sophismata</i> ⁸⁰	<i>Logica magna</i> ⁸¹	<i>Medulla dialectice</i> ⁸²
[1a] Item posito quod scias quod hoc sit Socrates vel Plato, nescias tu tamen an hoc sit Socrates nec scias an hoc sit Plato.	[1b] Quarto pono quod tu scias hoc esse Socratem vel Platonem, dubites tamen, an sit Socrates, et dubites, an sit Pla-	[1c] Tertio arguitur sic. Et pono quod hoc sit Sortes, quod scias esse Sortem vel Platonem, sed lateate an sit Sortes	[1d] Septima instantia. Hec est scita a te ‘hoc est Socrates’ et dubia, ergo contra conclusionem.

⁷⁷ See above for the relevant literature (fn. 52).

⁷⁸ The fourteenth-century debate extends beyond these examples, but these appear sufficient for the present purposes and were available in printed editions. To mention another important example, the slightly modified sophism is discussed in JOHN HUNTER 1999, 435. This is noteworthy, since Hunter appears to be one of *Logica magna*’s sources, as argued by Hughes in his edition of PAUL OF VENICE 1990, *ad indicem*.

⁷⁹ WILLIAM HEYTESBURY 1494, fol. 12vb (emphasis mine).

⁸⁰ PAUL OF VENICE 1493, fol. 51rb (emphasis mine).

⁸¹ PAUL OF VENICE 1981, 86 (the style of the quotation was modified to enhance coherence with other quotations, emphasis mine).

⁸² JEROME PARDO 1505, fol. 101rb (emphasis mine).

	to.	et lateat te an sit Plato.	
		Isto posito haec propositio ' <i>hoc est Sortes</i> ' est scita a te et eadem est tibi dubia, ergo scitum a te est tibi dubium. Patet consequentia et antecedens probatur:	Et pono quod dubites an hoc sit Socrates vel hoc sit Plato, scias tamen hoc esse Socratem vel Platonem (uno illorum demonstrato).
[2a] Et tunc erit ista propositio tibi dubia: ' <i>hoc est Socrates</i> '.	[2b] Et patet quod ista propositio: ' <i>hoc est Socrates</i> ' est tibi dubia.	[2c] Nam quod illa sit tibi dubia patet per casum.	[2d] Quod illa: ' <i>hoc est Socrates</i> ' sit tibi dubia patet ex casu.
[3a] Et quod ista sit scita a te, arguitur sic, quia tu scis illam significare precise sicut tu scis esse. Quod probo, quia tu scis quod illa significat quod hoc est Socrates vel hoc est Plato et tu scis ita esse	[3b] Sed arguitur, quod est scita a te, quia tu scis hoc esse Socratem vel Platonem, et scis quod ista sic adequate significat, ergo tu	[3c] Et quod illa sit scita a te probatur: Tu scis eius significatum primum, quod scis significari primarie per illam, ergo tu scis illam. Antecedens probatur: tu scis hoc esse Sortem vel Platonem, et hoc est significatum primum illius, quod scis	[3d] Sed quod sit scita, probatur, quia tu scis hoc esse Socratem vel Platonem et illa propositio significat hoc esse Socratem

<p>(scis enim quod hoc est Socrates vel quod hoc est Plato), igitur tu scis istam propositionem.</p> <p>Consequentia patet. Minor etiam patet ex casu.</p>	<p>scis istam.</p> <p>Patet consequentia cum maiori.</p>	<p>esse suum significatum primarium, igitur etc.</p> <p>Patet consequentia cum maiori per casum.</p>	<p>vel Platonem, ergo illa propositio est a te scita.</p>
<p>[4a] Et maior arguitur, quia tu scis quod illa precise significat quod hoc est Socrates et scis quod sequitur: <i>'illa significat precise quod hoc est Socrates, ergo illa significat precise quod hoc est Socrates vel quod hoc est Plato'</i>.⁸³</p>	<p>[4b] Et minorem declaro, nam tu scis istam: 'hoc est Socrates' significare adequate hoc esse Socratem, ergo tu scis istam significare adequate hoc esse Socratem vel Platonem.</p>	<p>[4c] Et minor probatur, nam haec propositio: 'hoc est Sortes' significat primarie hoc esse Sortem, ergo illa significat primarie hoc esse Sortem vel Platonem.</p>	<p>[4d] Minor probatur, quia significat hoc esse Socratem, ergo significat hoc esse Socratem vel Platonem.</p>

⁸³ Some copies of Heytesbury's texts have preserved a different, shorter version of the argument, which is even closer to *Logica magna*; as an example, see the following passage from one Leipzig manuscript: "Maior arguitur sic: tu scis quod ista: *'hoc est Socrates'* precise significat quod hoc est Socrates per casum, ergo tu scis quod ista precise significat quod hoc est Socrates vel quod hoc est Plato. Probatur consequentia: arguitur enim a parte disiuncti ad totum disiunctum sine negacione et sine distribucione," WILLIAM HEYTESBURY (ms.), Leipzig, Universitätsbibliothek, ms. 1360, fol. 112va (this diversity of *Regule's* textual tradition is discussed in HANKE forth.(1)). Note that both versions of the argument are described in the same way, but the 'short version' appears to be a closer fit to the description, as the 'long version' would require a more detailed description. Other examples can be disregarded in this study, but if Paul of Venice used Heytesbury's treatise as a direct source in this passage, he appears to have used the shorter version of the argument. However, John Hunter's version is close to the 'short version' too: "Tu scis istam significare primo hoc esse Johannem. Ergo tu scis istam significare primo hoc esse Johannem, vel Platonem," JOHN HUNTER 1999, 435.

<p>Arguitur enim a parte disiuncti vel a parte disiunctive ad totam disiunctivam sine negatione et sine distributione. Ergo etc.</p>	<p>Patet consequentia a parte disiuncti ad totum disiunctum, antecedens est concedendum a te, ergo et consequens.</p>
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<p>Patet consequentia a parte disiuncti ad totum disiunctum, et antecedens est verum, igitur et consequens.</p>	<p>Patet consequentia a parte disiuncti ad totum disiunctum.</p>
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All four arguments posit the scenario in which the agent knows that some person is Socrates or Plato, but is uncertain whether that person is Socrates and is uncertain whether that person is Plato. The key doctrinal difference between the different formulations is in rows [3] and [4]. Note that while the same rule (disjunction-introduction) is cited in [4a]–[4d], it applies to different moves. All four texts attempt to prove that the agent knows the sentence ‘*this is Socrates*’ in the posited scenario. In all four versions of the argument ([3a]–[3d]), the agent knows that the person in question is Socrates or Plato. In [3a], [3b] and [3c], the knowledge of ‘*this is Socrates*’, is derived from the knowledge that the person is Socrates or Plato and the knowledge that the sentence ‘*this is Socrates*’ signifies that the person is Socrates or Plato (which is true). As opposed to that, Pardo’s argument [3d] only operates with the knowledge that the person is Socrates or Plato and the fact that that is what the sentence ‘*this is Socrates*’ signifies, which could be due to weaker requirements on sentential knowledge.⁸⁴ More significantly, there is a split between [4a] and [4b] vs [4c] and [4d] in the supporting argument: while [4a] and [4b] attempt to prove that the agent knows that the sentence ‘*this is Socrates*’ signifies that the person in question is Socrates or Plato, [4c] and [4d] only attempt

⁸⁴ Another option is that something is missing from Pardo’s text, but the two editions are in agreement on this point.

to prove that this is what the sentence signifies. In other words, the key inferences are as follows:

[4a/4b] You know that '*this is Socrates*' signifies precisely/adequately that this is Socrates; therefore, you know that '*this is Socrates*' signifies precisely/adequately that this is Socrates or Plato.

[4c/4d] '*This is Socrates*' signifies (primarily) that this is Socrates; therefore, '*this is Socrates*' signifies (primarily) that this is Socrates or Plato.

For [4b], this move is not further justified, while [4a] assumes that the agent under scrutiny knows that an inference along the lines of [4c/4d] is valid, which allows the reduction of [4a] to an instance of the axiom **K**.⁸⁵ Either way, the passage includes sentences with considerably complicated logical structure, including the concatenation of two hyperintensional operators, which alone makes the argument open to the objection that it is not a straightforward instance of disjunction-introduction.⁸⁶

The original solution consisted in formulating certain principles of epistemic logic that would be sensitive to different forms of sentential meaning; ultimately, Heytesbury denied that the agent is aware of '*this is Socrates*', signifying that this is Socrates or Plato and even that this is the sentence's primary and principal signification.⁸⁷ The underlying reason seems to be that it is unrealistic to require that the agent is aware of the full meaning of a sentence if sentential meaning is closed under entailment. Two straightforward solutions to this problem are to propose a more realistic requirement, such as its restriction to 'primary' or 'explicit' meaning, or to abandon the view that the

⁸⁵ The aforementioned 'short version' is identical to [4b] in this regard.

⁸⁶ For the notion of hyperintensionality, see CRESSWELL 1975.

⁸⁷ "Ad quartum argumentum admitto totum usque ad hoc quod dicitur quod scio quod hec propositio '*hoc est Socrates*' significat precise quod hoc est Socrates vel quod hoc est Plato. Illam nego, nec illa primo et principaliter sic precise significat, sed primo et principaliter illa significat quod hoc est Socrates," WILLIAM HEYTESBURY 1494, fol. 15rb.

subject matter of propositional attitudes are sentences; as Heytesbury is (for whatever reason) unwilling to do the latter, he is prone to doing the former. The same part of the argument is attacked by Paul of Venice in *Sophismata*, with the exception that Paul rejects it in terms of a terminist analysis of hyperintensional contexts generated by the combination of the term ‘adequate’ with a hyperintensional verb ‘significat’.⁸⁸

In the *Logica magna* version of the argument, Paul of Venice denies that *primary* meaning is closed under entailment and notes that the inference in question is not an instance of disjunction-introduction by virtue of the presence of a modal operator.⁸⁹ This strategy is identical to the one endorsed in *Sophismata*, with the exception that it applies to a less complicated sentential context. Similar to Heytesbury, Paul of Venice continues by asking whether sentential meaning is closed under entailment; while he denies such closure to *primary* meaning generated by the terms of the sentence in this context, he is (surprisingly) open to that idea in general.⁹⁰

88 “Ad quartum respondetur consimiliter negando istam: ‘tu scis hanc: “hoc est Socrates” significare adequate hoc esse Socratem vel Platonem’. Et ad probationem: negatur consequentia, quia arguitur ad disiunctum stans collective ratione illius termini ‘adequate’ et termini concernentis actum mentis, sicut non sequitur: ‘iste terminus “homo” primo significat hominem, igitur significat primo hominem vel asinum’,” PAUL OF VENICE 1493, fol. 51vb.

89 “Ad tertium argumentum respondetur concedendo quod illa ‘Hoc est Sortes’ est mihi dubia. Et cum probatur quod illa est scita a me, quia scio hoc esse Sortem vel Platonem et illa significat primarie hoc esse Sortem vel Platonem, nego quod illa significat primarie hoc esse Sortem vel Platonem, sed significat primarie hoc esse Sortem, ex quo non sequitur quod significat primarie hoc esse Sortem vel Platonem. Et si arguitur: ‘...a parte disiuncti ad totum disiunctum’, respondeatur ut prius in proximo argumento,” PAUL OF VENICE 1981, 94. The passage he seems to refer to is: “Et tunc ad argumentum: ‘tu dubitas propositionem falsam, igitur dubitas propositionem falsam vel aliquid esse non aliquid’, nego consequentiam. Et cum dicitur: ‘...a parte disiuncti ad totum disiunctum’, dico quod non valet consequentia quando modus praecedens disiunctum determinat sicut est in proposito,” PAUL OF VENICE 1981, 92.

90 “Ita in proposito dicatur quod sive illae convertantur sive non convertantur materialiter vel formaliter : ‘Hoc est Sortes’ et ‘Hoc est Sortes vel Plato’, significatum primum unius non est significatum primum alterius, quia significatum primum primae est significatum simplex ortum a simplicibus extremis, et significatum primum secundae est significatum disiunctum ortum a terminis compositis, ut patet. Non tamen nego quin si-

Pardo's solution to the sophism has two notable features. First, similar to his predecessors, he takes it as an opportunity to address sentential semantics. As opposed to Heytesbury and Paul of Venice, who are primarily concerned with the formal-semantic aspects of the problem, Pardo mentions the ontological aspect of the problem. He introduces two alternative views of sentential meaning or '*complexe significabile*', which were discussed in preceding parts of *Medulla dyalectices*.⁹¹ However, the distinction plays no significant logical role in the solution to the sophism: Pardo translates the problem into the terminist framework by introducing appellation to explain the restrictions blocking the opening argument and demonstrates its function by distinguishing between *de re* and *de dicto* contexts generated by the verb 'signify'.⁹² It seems possible to label Pardo's approach as a Continental solution to a British problem. Originally, the problem consisted in the consequences of Bradwar-

nificatum unius significetur a reliqua et econtra, sed non primarie," PAUL OF VENICE 1981, 98.

91 For an overview of the debate on *complexe significabilia* in the post-medieval period, see NUCHELMANS 1980, 45–73 and (among her other works), ASHWORTH 1978, 81–121, ASHWORTH 1981, 61–96, and a recent analysis of Pardo's position is presented in PÉREZ-ILZARBE 2016, 512–531 (who defended a doctoral dissertation on Pardo's sentential semantics).

92 "Ad hoc argumentum brevis patet responsio. Nam si ponantur complexe significabilia, ut ponit primus modus dicendi, illa est falsa: '*illa propositio "hoc est Socrates" significat hoc esse Socratem vel Platonem*', neque valet consequentia: '*significat hoc esse Socratem, ergo significat hoc esse Socratem vel Platonem*'. Quando enim arguitur a parte disiuncti ad totum disiunctum, servandum est ut reliqui termini respectu quorum non arguitur a parte disiuncti ad totum disiunctum pro eodem teneantur in antecedente et in consequente, quod non est in proposito. Nam quando dico: '*hec propositio "hoc est Socrates" significat hoc esse Socratem*', *ly significat* appellat significationem istius '*complexe significabile hoc esse Socratem*', sed cum dicitur in consequente '*significat hoc esse Socratem vel Platonem*', *ly significat* appellat significationem illius '*complexe significabile hoc esse Socratem vel Platonem*', ideo *ly significat* non eodem modo stat in antecedente et in consequente. Si etiam teneatur secundus modus de complexe significabilibus, tenendo restrictionem illam rigorosam dictam negaretur etiam illam consequentiam esse bonam propter variationem appellationis de *ly significat*. Ex isto antecedente potest inferri istud consequens: '*ergo hoc esse Socratem vel Platonem significat*'. Ultra tamen illa potest dici quod illa propositio: '*hec propositio significat Socratem vel Platonem*' potest distingui: aut est disiunctiva, et sic est vera, aut de disiuncto extremo, et sic est falsa, tenendo restrictionem supradictam," JEROME PARDO 1505, fol. 101rb.

dinian semantics for Heytesburian epistemic logic. The problem would later become further complicated by introducing propositional ontology, but that did not, ultimately, influence the general strategy. As such, Pardo's approach exemplifies a more general trend which Ashworth, in the context of the analysis of intensional and hyperintensional verbs (such as 'promise' and 'require'), described as preferring Buridan to Heytesbury.⁹³

5. Conclusion

The analysis of epistemic sophisms discussed in John Mair's circle contributes to the exploration of sixteenth-century Parisian logic. The general trend can, in this particular case, be summarised as 'British logic mediated by Italian commentators'. While the original source for most of the problems discussed above was William Heytesbury, the actual sources for John Mair's circle appear to be Paul of Venice and Cajetan of Thiene.

There are two notable terminological peculiarities pertaining to the Parisian authors as compared with their British and Italian sources. First, there is Pardo's use of *appellatio* in his solution to the problems associated with Bradwardinian semantics. Second, there is a common tendency to replace belief or knowledge with assent.⁹⁴ Both tendencies appear to be instances of bringing Continental tools into British debates. Furthermore, there is an interesting trend for naturalising logic and epistemology, such that the solution of some of the sophisms becomes an empirical (and, in some cases,

⁹³ See ASHWORTH 1974(1), 148 and ASHWORTH 1976, 71-72.

⁹⁴ As an example, this is Heytesbury's working definition of knowledge: "...scire non est aliud quam sine hesitatione apprehendere veritatem, idest credere sine hesitatione quod ita sit et cum hoc quod ita sit ex parte rei..." WILLIAM HEYTESBURY 1494, fol. 13vb. In contrast, Pardo used the following formulation of the same problem: "Uno modo accipitur scientia pro assensu propositionis vere vel melius illius significati sine formidine de opposito..." JEROME PARDO 1505, fol. 96ra. For the relations of such formulations to the fourteenth-century Parisian tradition, see LAGERLUND 2019, *passim*.

quantifiable) problem.

This paper has some loose ends. The analysis of the three bodies scenario showed a common distortion of the original argument and what appears to be a mis-reference to Cajetan of Thiene. This is suggestive of either missing sources or covert connections. The first problem could be solved by broadening the corpus, assuming that such a source or group of sources was a written text rather than, say, an undocumented lecture attended by one of the authors, and was not lost. The second problem could only be solved conjecturally based on biographical and institutional data. As a hypothesis, such a mis-reference could have originated between Mair and Coronel and was first published by Coronel; one of these two could also have been responsible for its further dissemination.

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