The Growing Block and the Problem of the Continuum*

Shira Yechimovitz
Tel Aviv University

Shira Yechimovitz is an M.A. Student at the Cohn Institute for the History and Philosophy of Science and Ideas, Tel Aviv University. Her research currently focuses on the growing block as a hybrid A-B theory, in particular temporal order and direction in the theory.

The orthodox approach to time states it to be a continuum. In this paper I aim to show that the growing block model poses a unique problem to the continuity of time, on account of it being a hybrid A-B-theory. Tension lies in the fact that a continuous B-theoretical block is built through the A-theoretical becoming of instantaneous slices of present. First, I show that a continuous growing block necessitates a present with zero temporal duration; second, I show that such notion of present rules out some widely accepted B-theoretical solutions to the problem of the continuum, while its commitment to the B-theory rules out some of the A-theoretical ones. Finally, I will discuss possible consequences.

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BECOMING CONTINUUM TEMPORAL DIRECTION
TEMPORAL EXTENSION GROWING BLOCK

I. Introduction

The standard conception is that time and space are continuous. Since time is commonly considered structurally similar to a one-dimensional space, the time continuum is often portrayed as the Euclidian straight geometrical line. Just as the straight line is made up of extensionless points, the time continuum is taken to be constructed of point-like durationless instants. The continuum is gap free: it persists without any breaks or interruption; it is dense: between each two point there is always another point; and it is infinitely and indefinitely divisible: any division of the continuum will always result in parts that can in turn be further divided, without ever resulting in indivisibles (Bell 2019, viii). These properties of the continuum on the one hand, and its being constituted by points on the other, raise challenges that were the subject of a long lasting philosophical and mathematical inquiry. The tension increases when it comes to time.

A geometrical line is not a full description of the ordinary (common-sense) notion of time, since it lacks two important unique features. The first is that time does not only have an order, it also has a preferred direction: it runs from earlier to later. The temporal series that runs from earlier to later is known as the B-series, and the relations between its terms will be referred to as B-relations. The second feature of time is that it appears to involve some sort of flow or change. It seems as if every instant in time changes from being future, to being present, to being further and further in the past. The series whose terms are past, present, and future is called the A-series (McTaggart 1908).

B-theories of time embrace the B-series and reject the objectivity of the A-series. They entail a space-like time and a metaphysical picture of the universe as a static four-dimensional spacetime block. Such views are compatible with the currently accepted Cantorian account of continuity. A-theories argue that the A-series is objective and things in time do change in respect to its terms. A-theories entail a dynamic metaphysical view and are therefore more inclined to embrace different accounts of continuity. Finally, the growing block view is a hybrid of the two. It is committed both to the view of the universe as a four-dimensional spacetime block that is eternally fixed in B-relations, and to the claim that there are objectively distinct A-categories, in respect to which time changes.

According to the growing block theory, the change of time itself cannot be put in the same terms as change in things in respect to time. An illustration of the latter is the change Tom Smith undergoes as he grows taller than his father John. There is a time *t* in which the son is shorter than his father and a time *t'* in which he is taller than his father. The change of things in time is relative to time. The change of a moment in time is different, and can be compared to the change Tom Smith undergoes as a new child is born into the family. If Tom is John's youngest son, he will remain so until the birth of a new sibling. Afterwards, Tom will no longer be John's youngest son. A change has occurred, yet Tom himself did not undergo any intrinsic change. What changed is the fact that a new entity came into being, thus forming new relations with the pre-existing members of the family. Such is the change of time. Three-dimensional slices of reality keep coming into existence, joining the world history one by one, thus forming a four-dimensional growing block. The future does not exist. The present is the most recent slice that came into being, and once another slice is added, it becomes the new present and the precedent slice becomes past. This process is called becoming (Broad 1923, 65-68).

The growing block's dual nature makes the puzzle of the continuum

seemingly impossible to resolve, as the block grows through the becoming of three-dimensional slices with zero temporal extension. Or if we think of time as a straight line, time in the growing block universe literally is formed through the accumulation of point-like instants. In the following section I show that the present slice cannot have any non-zero temporal extension, and that the block can only grow through the becoming of one such durationless slice at a time. In the third section I lay out some of the problems the point-like present brings about and their accepted solution. I then show why they do not apply to the growing block. In the final section I discuss possible consequences and conclusions.

II. The Growing Block and the Problem of the Extended Present

When holding to a continuous growing block view, a point-like present is inevitable. The only intelligible option for the growing block theorist is to admit that the temporal dimension of the block is constructed through the addition of present slices under two restrictions: (a) the present cannot have any non-zero temporal extension; and (b) the growth of the block can only occur through the becoming of *one* such instantaneous slice of reality at a time. These restrictions are a direct consequence of two pillar notions of the growing block theory. The first is the very definition of the A-categories in the model, which entails that the present moment cannot have any non-zero duration; the second pertains to the forming of the B-relations through the process of becoming, which cannot possibly allow for any duration of time greater than zero to become all at once.

II.1. The Problem of the Defining Features of the Present

The A-theoretical commitments of the growing block entail an objective present moment that is distinct from the two other temporal categories, the past and future. This distinction is derived from what Miller calls «the growing block ontological thesis»; namely, that the past and the present exist, while the future is a non-existence (Miller 2013, 348). The present moment is topologically distinct from the rest of the block by virtue of being the only slice of existence that has no successor: «the essence of a present event is not that it precedes future events, but that there is quite literally nothing to which it has the relation of precedence» (Broad 1923, 66). But this is not the full picture; the growing block's A-theoretical commitments also include what Miller calls «the dynamic thesis», according to which the answer to the question about which moment is the present is ever-changing (Miller 2013, 348). This change is possible due to the process of becoming; namely, the coming into existence of one temporal slice at a time. As a new slice comes into existence on the edge of reality, the slice that was once present becomes the past and goes deeper into the past as more slices accumulate one by one in a gradual process. The process of becoming entails not only the ordering relation of earlier and later than, or *succession* – it also entails enumeration, or *consecutiveness*, meaning the instantiation of the temporal particulars one by one and therefore one before the others (Craig 2000, 234-235). Hence the very definition of the present in the growing block consists of two distinct features: one, it has no successors; and two, it is not just the *last* temporal slice in the ordered, directed block, but is also the *latest* addition to reality. But these two defining features of the present cannot be consistent with the notion of extended present.

Let us assume a continuous growing block in which the present has duration greater than zero. Each new slice would then have some temporal extension,

even if very small. For convenience, consider such an extended temporal slice as a temporal interval on the growing block. Any such temporal interval of the growing block would then have to be indefinitely divisible into smaller temporal intervals that in turn are also indefinitely divisible. Since those sub-intervals are all parts of the latest addition to reality, each of them would also be considered a present slice. As a result we will end up with an entity that is contradictory by its very definition: a "present" slice that has one or more successors. And since one of the properties of a continuous interval is being indefinitely divisible, any such "present" slice could always have infinitely many successors, and no amount of division will ever lead to a single present slice that has no successors.

A possible objection might be that while this line of argument might enforce restriction (a), it does not necessarily imply that restriction (b) has to hold as well. Indeed, the objectors could say that it is easy to see why the *present* has to be point-like, but then ask: must the block grow through the becoming of one duration-less slice *at a time*? In line with this objection, some might try to reconcile the conflict between the extension of the temporal slices and the defining features of the growing block's present by appealing to what Forbes calls «past genesis» (Forbes 2015, 191). In the context of the growing block, past genesis refers to things in time coming into existence directly as past, without having been present first. If the present is the edge of reality and the past is merely anything that precedes the present – the supporters of past genesis might say – there is no reason why the block should not grow in extended temporal intervals made mostly of past with one unique present end-point; namely, the boundary between this new temporal interval and nothingness. Thus, the proponents of past genesis attempt to hold on to the defining features of the present, while avoiding the problems of the point-like present and the continuum (to be discussed in section III). It might seem tempting to turn to past genesis as a reply to the objections the problems of the continuum may give raise to, but there are good reasons not to. For one, it strengthens another objection to the growing block known as the epistemic problem.

The epistemic problem, also known as «the present problem» (Bourne 2002) or «the now-now problem» (Braddon-Mitchell 2004) might be summarised as follows: we believe that our time is the present, but granted a growing block metaphysic, we are much likelier to be in the past. If we embrace (a) but not (b), not only do we have no way of knowing whether our time is the true present, but the hope that we ever *were* truly in the present is faint. Past genesis also weakens the dead past growing block: a variety of the growing block that was conceived in response to the now-now problem. (Forrest 2004, Forbes 2016) According to dead past growing blockers, we can know with certainty that we are in the present moment, since only this very instant can sustain activity or processes and consciousness. The misfortune of the past that was born dead is more than just grim; past genesis disqualifies the dead past growing block's line of defence against the epistemic objection, as well as against the problem of past truth-makers, in particular the resolution to the problem suggested by Forbes (2016), that relies on the fact that things in the past used to be present.

Setting aside the epistemic objection and the dead past solution, there is a straightforward, conclusive way to prove past genesis to be impossible in a growing block universe, as past genesis ignores the second feature of the growing block's present; it is not only the *last* moment, but also the *latest*. This will become clearer in the following discussion of the way the block's B-relations are formed.

II.2. The Problem of Consecutiveness in Becoming and the Forming of B-Relations

As proponents of the A-theory observed, the process of becoming involves not only succession, i.e., the B-relations of earlier than and later than, but also enumeration, or consecutiveness, meaning the instantiation of the temporal particulars one by one and therefore one before the other (Craig 2000, 234-235). In the growing block view, the notions of temporal succession and consecutiveness are intimately connected. The succession of temporal slices depends on the consecutiveness of their becoming, which makes it impossible for a single temporal slice to have internal succession. Like the B-theories, the growing block is committed to the thesis that everything that exists in time does so in eternally fixed B-relations. Different B-theorists may employ different strategies to account for the order and direction of the time series. Some will take them to be primitive; others, as the result of asymmetries in the contents of time or in our consciousness, rather than features of time itself. But the growing block also adheres to the commitment that the temporal order and direction of the block are not arbitrary or illusory; rather they are objective features of time itself. It is widely held that in contrast with a static B-theory, objective temporal becoming has the ability to account for temporal direction (Craig 2000, 256-258), and the growing block makes use of this notion of becoming for this very purpose.

In other words, very much like in McTaggart's original argument, in the growing block model, the B-series stems from the A-series. When a new slice of reality is added through becoming, new relations between this slice and the sum total of reality come into existence that did not and could not have existed before, since before it came into being that slice was nothing at all (Broad 1923, 66). That includes the relations of temporal succession. To put it more simply, the fact that slice S at time t is later than slice S' at time t' is the direct consequence of S' coming into existence *after* S already existed.

It now becomes clear that the growing block does not allow for the present to have any non-zero duration. The B-relations of earlier than and later than in the growing block are formed through becoming. As a temporal slice comes into existence through the process of becoming, it concurrently becomes the immediate successor of the slice that was formerly the edge of being. Thus, any amount of existence that joins the universe through a single act of becoming can only be simultaneous in respect to the temporal dimension, which means all of the content of such a slice necessarily occupies a single temporal location in the B-series. Obviously, for anything in the spatiotemporal block to have duration – whether we refer to it as an event, a temporal slice or an interval of time is immaterial – it would have had to be spread across several B-locations, and there would have had to be a distinction between its earlier and later parts. But as previously shown, anything that comes into being together is B-simultaneous, and therefore cannot be divided into distinguishable, successive parts. The contents of each new slice are always simultaneous and therefore it takes more than one slice to form a temporal segment whose duration is greater than zero; whenever there is more than one slice, it means their becoming occurred consecutively, or else they would have been one and the same slice and therefore have a single B-location and zero duration. Thus, there can only be one new durationless slice at a time, and only one such slice can be the latest addition to reality. The derivation of succession from consecutiveness renders both the extended present and past genesis as impossible for the growing block.

III. The Point-Like Present and the Continuum

In this section I will explore two ways in which the point-like present may pose a challenge to the time continuum in the growing block model; the first is the problem famously known as Zeno's paradox of plurality: how can durationless slices add up and form temporal duration (zero times infinity still equals zero); and the second is that one of the properties of the continuum is density, yet it is unclear whether a growing block metaphysics allows for the time series to be dense.

III.1. The Problem with Point-Like Presents Forming Duration

The fact that the growing block builds through the accumulation of durationless slices poses a twofold problem. On the one hand there is the problem of addition; clearly the block itself has temporal duration, but it consists of individual slices that have zero duration, and even infinitely many such slices could never amount to any duration greater than zero. On the other hand there is the problem of temporal locations; the present slice lies at the very edge of the block, and when a new slice is added through becoming, it must also come to be exactly on the very edge of the block, or else the block could not be gap-free. In that case, how can B-locations be distinguished one from the other? Let the edge of reality be in time t_n. As long as the slices are actualized on that very location, there can be no further locations; in order for the location t_{n+1} to come into existence, a temporal interval from t_n to t_{n+1} will have to become all at once, which I showed to be impossible on the growing block. This applies equally to any number between n and $n+1$, no matter how small. Given that the present is point-like, the temporal location of each and every new slice would be the same as of its predecessor, and consequently, as of the entire block.

The solutions I am about to discuss in this section have one thing in common: they discredit the underlying assumption that the length of an interval on a line is generated by adding up the lengths of its smallest continuants. Looking back to the growing block account previously given, it becomes evident why proponents of the model might want to hold on to such an assumption.

The first solution is to view the line as prior to the points. This approach was first introduced in Aristotle's discussion of Zeno's paradoxes and later embraced by Brentano, Peirce and others. (Bell 2019, 157, 163). Continuous magnitudes are potentially divisible to infinity in the sense that they may be divided anywhere, though they cannot be divided everywhere at the same time. Aristotle's argument is as follows: first, since points have no parts, they cannot form a continuum, and second, the continuum is prior to its parts and points exist not in actuality, but as limits of lines (*Phy.* 231a21-b10). The continuum has the potential to be infinitely divided, not into points, but into segments that can also be infinitely divided, and so on, and so on. The points are the boundaries of the segments that are the parts of the continuum, but any such parts only come into being as the wholes are divided.

But this cannot be said of the growing block, seeing that addition in fact occurs in the process of becoming. Given a growing block metaphysics, *there has to be* such a thing as an actual instantaneous slice of time, and as I showed the block cannot exist prior to these slices and can only grow through the becoming of such slices. We cannot say that the time continuum is prior to these temporal slices, because nothing would have existed in time if it were not for the constant accumulation of these slices. Thus, if we are committed to the growing block view we must admit them to be *actual entities*, the components of the block and not just the potential

boundaries of its potential parts. Moreover, the problem of locations still remains as they cannot exist prior to the becoming of their content.

Let us move on to an approach that might allow for the instantaneous slices constituting the block to be actual. Our best science takes the Cantorian approach to the continuum, which models the continuum after the real number line. Cantor was able to prove that any finite segment on the real number line – regardless of how short or long – contains the same number of points as any other segment on the real number line, and as the entire real number line. I will briefly present the Cantorian account, and then show how it resolves the paradox of plurality. Then I will show why the Cantorian solution cannot be applied to the growing block.

Cantor proved something that seems counter-intuitive: that there are different sizes of infinity. An infinite set can be either denumerable (countable) or non denumerable. An infinite set is denumerable if it can be ordered in such a way that each of its members has finitely many predecessors. An example of a denumerable set is the set of integers. All denumerable sets can be put in a one-to-one correspondence with the set of positive integers, which is neither gap free nor dense. Cantor was able to establish a one-to-one correspondence between the set of integers and the set of rational numbers, which is dense, but not gap free. So despite the fact that there are more rational numbers than integers on finite segments of the same length, both infinite sets have the same number of members. But as Cantor proved, the set of real numbers cannot be put in a one-to-one correspondence with the integers. Furthermore, any proper subset on the real number line can be put in a one-to-one correspondence to any other proper subset on the real number line, and to the real number line itself. Hence, any segment on a continuous line, no matter how long or short, contains the same number of points as the entire real number line (Dainton 2010, 277-280). Therefore, the extension of a continuous line does not depend on the points it consists of. In everyday life, we take the size of objects to be the sum of the size of their smaller parts. If a segment of the sidewalk consists of paving stones, we would be right to think of the segment's length as the sum of the lengths of those paving stones. But since any segment on a continuous line contains the same number of points regardless of its length, we would be wrong to treat it the same way we treat everyday objects.

But what other ways are there to determine the length of an interval if not by adding up its points? One way available in mathematics is called "measure theory". According to measure theory, an interval must contain non-denumerably infinite amount of points for it to have length. The length is the distance between the interval's end points. Since length is not an intrinsic feature of a set of points, the points themselves do not matter. The addition of a single point to an interval has no effect whatsoever on its length. Instead, the line must be associated with a metric or distance function (see Dainton 2010, 281-283). But if we try to apply this answer to the growing block, we will run into a problem. The sum total of reality at any time tn consists of slices $S1+S2+...+Sn$. Let us say the temporal duration of the universe as of time tn is not the sum of the duration of the temporal slices forming it (which has to be zero). Reality then increases through the becoming of a new slice $Sn+1$ at a the new time $tn+1$, which are also durationless. So how can the duration of time as of $tn+1$ be greater than the duration of time as of tn? Even if we assume that as of t_n there already was a pre-existing four-dimensional block with temporal extension, we cannot say that the adding of $Sn+1$ to the block makes it grow in respect to its temporal duration. And if we assume there is a starting point to reality such as the big bang, there will be no temporal duration at all. Another thing is that according

to measure theory, an interval must contain a non-denumerably infinite number of points in order for it to have measure, which leads us to the problem of density.

III.2. The Problem of Density and Becoming

Euclid defined the straight line as a length without breadth, and if the line is composed of points, we take that to mean that there are infinitely many points between every two points on the line. Namely, that instances in time form a dense series:

> Between every two elements of a dense series there will be at least one and therefore an infinity of other elements; so that no element has a successor, and no element a predecessor (Huntington 2003, 34)

As this definition makes very clear, a consequence of density is that a member of a dense series cannot have an immediate successor or predecessor. This contradicts one of the key commitments of the growing block view. Once a temporal slice is added to the sum total of reality through becoming, it comes into relations with the rest of reality. These relations, once formed, are eternally fixed (Broad 1923, 69). So once a new slice becomes on the edge of reality, a relation of succession is formed between it and the previous slices. Thus, every point on the time series has one direct predecessor and every single *past* slice has one direct successor. Once a slice's immediate successor comes in to being, there is no way of coming up with infinitely many slices in between them as density requires. In other words: between two different elements of the growing block time series, there can only be a finite or zero number of elements.

Once again, it is the combination of the metaphysical commitments of the B-theory and those of the A-theory that keeps the growing block theorist from achieving resolution. The fact that the B-series of time is formed through consecutive A-theoretical becoming conflicts with the property of density. The B-theorist can still accept that the indefinite divisibility will never lead to indivisibles or that there are a non-denumerably infinite number of points composing the continuum. But this means giving up completely on any A-theoretical notions. Such a block can never be formed through becoming.

III.3. Pure A-Theoretical Solutions

As previously mentioned, the B-theories of time need not be threatened by the notion of the point-like instant, and more specifically the point-like present. B-theoretical time is space-like and so B-theorists are not committed either to the objectivity of the present moment or to its dynamicity. Thus, the B-theorists are free to embrace the Cantorian continuum or the notion that points are potential or ideal. So perhaps the answer to the growing block's problem can be found in the A-theoretical solutions to the problem of the continuum. The bind is that the doctrine of the instantaneous present is incompatible with becoming (Craig 2000, 236). A pure A-theory is open to endorsing different solutions. Thinkers like Bergson, Brouwer, Wyle, and others assert that the numbered, mathematical notion of the continuum is not how time is in itself. The true nature of time is given to us by our intuition. Those who hold these views are free to assert that all that exists is a completely unified and indivisible duration of time and accept a primitive pre-metrical notion of the present. Could the proponents of the growing block too?

A-theoretical becoming cannot endorse an instantaneous present without raising Zeno's paradoxes of plurality and motion. One possible solution is adhering to the view that there is no such thing as the present *simpliciter*. The answers to question such as "what is the present time?" or "how long is the present time?" vary depending on the context in which they are asked. "Present time" can mean the present second, the present hour or the present decade (Loizou 1986 in Craig 2000, 248). This allows for duration to be taken as prior to metrics.

The growing block may hold to a dynamic thesis regarding the present moment, but is also, in a way, committed to a static block; new things come to be constantly, but once something enters existence, it remains unchanged (Broad 1923, 66, 79-84). Broad declares that it is misleading to call becoming "a change" (Broad 1923, 68), which is one reason it cannot fit with the idea expressed as Bergson's *durée réele*, or Prior's idea that there is no transition of instants from being future to being present to being past: the present time is all that exists and the change is in things, or as he puts it: «the basic reality is things acting» (Prior unpublished, in Craig 2000, 246). By accepting a primitive pre-metrical notion of the present, A-theoretic metaphysics can avoid Zeno's paradoxes (Craig 2000, 248). But the growing block cannot accept such notion. Perhaps the most evident incompatibility of the growing block with such conceptions of temporal continuity lies in the fact that the growing block is committed to the claim that there is a temporal dimension along which all of the events in time are ordered and in respect to which reality keeps expanding. Such ordered homogenous space-like time is in complete opposition to these notions of duration and continuity.

IV. Conclusion

I showed that in a continuous growing block universe, the present has no duration. This is a direct result of its being a hybrid A-B-theory. The fact that the growth of the block can only occur through temporal slices of zero extension conflicts with the thesis that time is a continuum. All the B-theoretical solutions for the problems of the continuum that I tested so far conflict with the A-theoretical elements of the growing block, and vice versa. But the growing block is still a very intuitively and metaphysically appealing theory, so it would be beneficial to find a way to settle the problems of continuity. One approach is to continue looking for other accounts of continuity. Another is to reconceive the growing block in such a way that can transcend the conflicting traits. Or it could be that the growing block spacetime is inherently discrete.

IV.1. Revising the Orthodox Continuum

The Cantorian view of the continuum is not without its weaknesses, which gave rise to (roughly) two groups of opposing views: the hyper-dense Peircean continuum and taking extension as fundamental. Perhaps one of them could be a better fit for the growing block.

The idea underlying the hyper-dense continuum is that the continuum cannot merely be a collection of infinitely many points. In order to genuinely form a continuum, these points must be welded together, which cannot be conceptualized using the analogy of the real number line, as the true continuum requires an even greater number of points: nothing less than the maximal possible number of points. When packed in this hyper-dense manner, the points lose their individual identity and become welded together into a unified continuum. (Bell 2019, 163; Dainton 2010, 306).

While this view was proven useful in solving some of the puzzles of the continuum (see Dainton 2010, 307-309), it is hard to see how hyper-density could be applied to the growing block view given the problem of regular density and becoming discussed in section III.2. Moreover, since the temporal slices come into existence one by one, it is hard to see how they can ever reach such maximal quantity.

Extension as fundamental is the notion that the most fundamental parts of the continuum have extension, no matter how small. True, the orthodox continuum also has the property of being infinitely and indefinitely divisible, but it is also conceived as having indivisible, extensionless parts – i.e. the points that constitute a straight line – and it is this that the difference between the views hinges on. To take extension as fundamental is to abandon the idea that the points are the most fundamental constituents of a line. Instead, the parts of a line are lines, which are made of smaller lines, and so on without ever reaching a bottom level (Dainton 2010, 309-310). Taking extension as fundamental can provide a solution to the paradox of plurality, but it is incompatible with the growing block because – as I showed in section II – there must be such a bottom level in the form of instantaneous slices from which the block is constructed.

IV.2. The Growing Block Revised

In "A Reply to My Critics", Broad himself deals with the problem of the instantaneous present and suggests that time could have another dimension along which slices can be extended. Think of things in time as ordered along a T-axis, which stands for ordinary time. A temporal slice shall be represented as a point t on the T-axis. But if we add a Θ-axis, which stands for an additional temporal dimension, the temporal slice would be represented by a straight line of finite extension, parallel to the Θ-axis. (Broad 1959, 769-772)

Recall that the problem this paper is concerned with is that the growth of the block cannot occur through the becoming of durationless instants, yet at the same time it must thus occur. On the above suggestion, the temporal slice is T-instantaneous, but has Θ-duration, so one might say it succeeds in being both instantaneous and extended. However, I do not see any way how incorporating Θ-duration can help the growing block overcome the problems described in section III. More over, while there appears to be no inconsistency on this particular account, accepting a second temporal dimension still seems like a slippery slope leading to infinite regression or circularity.

There might still be a way to loosen the clutch of some of the commitments by adhering to a theory of growing events, rather than growing block. Perović proposes a theory of growing events that still holds to the same ontological and dynamic theses as the growing block. But the growing events theory departs from the growing block in that in the former, events are the most fundamental ontological constituents of reality (Perović 2019, 19). So the continuous four-dimensional block is but an abstraction form existing events, and so are the instantaneous slices:

> [...] it is not such slices with their instantaneous properties that build up the GE [growing events] theorist's events; rather, events are metaphysically prior and instantaneous slices and their properties are abstractions from events. This is just another way of saying that events take some time to unfold and such temporal extendedness of an event is difficult (if not impossible) to recover from instantaneous temporal slices of objects and their properties. (Perović 2019, 20)

On this account the universe grows, but it is a growing event rather than a growing block, and it consists of accumulating events, rather than slices. It appears the growing event theorist might be able to adopt a different, more A-theoretical account of the continuum. Perhaps this is a sacrifice worth making in order to preserve the ontological and dynamic theses of the growing block, because it may very well be that they are the source of the theory's strong intuitive appeal, and the block and slices are not indispensable features, but – some might say – redundant and even disqualifying. On the other hand there is the worry that these two theories differ on such fundamental grounds, that the growing events theory is not a defence of the growing block, rather it replaces it. Perović also notes that in order to keep to the growing block theory's original commitment to the privileged present, a variant of the growing event needs to be constructed that identifies the present not with "ongoingness" but with the very edge of being (Perović 2019, 22) and the question remains: how can the growing event theorist have an "edge of existence" and still avoid the B-theoretical succession?

IV.3. Can the Growing Block be Discrete?

The properties of the discrete and the properties of the continuous are diametrically opposed. While the continuous temporal interval is indefinitely divisible, the division of any discrete duration of time ends in indivisible atomic quantities called chronons. Chronons are usually defined as a certain minimal physical quantity. Indeed, when considering a discrete account of the growing block, questions from the physical sciences arise. While some of our best science relies on the continuity of time, quantum mechanics might support a discrete spacetime, and there are still some very promising theories, such as quantum gravity, according to which time might be discrete (Dainton 2010, 300-301; Rovelli 2018, 54-56).

But it seems that the theories in question do not concur with a growing block metaphysic. They might entail a dynamic metaphysical picture, but one that could not be farther from the growing block. They do not incorporate the notion of objective present, or the commitment of the growing block to temporal order and direction. As Rovelli puts it: «Time has loosened into a network of relations that no longer holds together as a coherent canvas» (Rovelli 2018, 58). Reality possesses no fixed, objective temporal relations or direction, and «in the vast universe there is nothing that we can reasonably call "present"»(Rovelli 2018, 59).

On the other hand, the special theory of relativity (which supports a continuous spacetime) and the growing block are far from a perfect fit. The special theory of relativity poses a challenge to both the ontological and dynamical theses of the growing block, and certainly to the notion of the objective present. (Miller 2013, 352-353). So physics gives us evidence *contra* the growing block, be it continuous or discrete. Evidence in support of the growing block may turn up in the future (more likely in the form of philosophical arguments, rather than scientific observations). In the meantime, there is still value in investigating other consequences of a discrete growing block picture.

A clear advantage of taking space and time as discrete is that, on this view, they can have a metric and they can be seen as formed of their smallest constituents. So perhaps if the growing block is discrete, there need be no tension in the fact that its temporal duration is constructed through the becoming of temporal slices. If the slices could be temporally extended, rather than instantaneous, we will be able to overcome the problems of addition and locations mentioned in section III.1. But

before we can arrive at that conclusion, the arguments against the extended present from section II must be rejected for discrete time, or else we end up back in square one (or in the first singular instantaneous slice of existence, if you prefer) stepping right into Zeno's paradox of plurality. But maybe by accepting the doctrine that time is discrete, the growing block theorist can salvage the extended present. That way, the duration of the entire block could be additive.

Thinking back to the defining features of the growing block's present, as long as the present slice cannot be divided into further present slices, there will be no paradoxical entities such as "present" slices with successors, and the latest addition to reality could be an atomic slice. But what will be of our commitment to the B-series? Can such an account give rise to Zeno's paradoxes of motion? Not necessarily. The growth of the block is not a kind of motion and must not be mistaken for motion (Broad 1959, 766-767), so there is no need to explain how the block grows from t_n to t_{n+1} without ever traversing infinitely many locations in between. There is nothing paradoxical about temporal slices just coming into existence on the edge of the block, as it is not the traverse of the block's edge from one temporal location to the next. The slices just become and by the consecutiveness of that becoming they can only become on the very edge of reality. And the fact that every other type of change in the growing block is reducible to becoming (Broad 1923, 67) does not conflict with the fact that once they come into existence, the slices remain static. Any motion of objects across space in the growing block can be analysed in a completely B-theoretical way such as Russell's "at-at" theory.

I trust that this line of thought has the readers of this paper warming up to the idea of the discrete growing block. But some questions still remain. What is the duration of the slices and how is it determined? Will they still be uniform once given duration? Perhaps the answer is that the features of the present slice entail that the extension of a single slice can only be as great as the extension of the smallest possible lapse of time: the chronon. The only thing that can be uneven is the qualitative difference between two slices, namely differences across the dimensions of space; any duration greater than a chronon is spread between more than two different B-locations, and therefore – by the way B-locations are formed – consists of more than a single slice. But what keeps them from being shorter than a chronon? There are still voices in the discussion who doubt the whole notion of chronons, claiming it to be incoherent and irrelevant (see Craig 2000, 240-242). But it seems that if chronons do exist, their size will be uniform and determined. In the words of Lee Smolin:

> According to loop quantum gravity, space is made of discrete atoms each of which carries a tiny unit of volume. In contrast to ordinary geometry, a given region cannot have a volume which is arbitrarily big or small – instead, the volume must be one of a finite set of numbers (Smolin 2000a, 106 in Dainton 2010, 300).

In conclusion, as a hybrid A-B-theory the growing block poses a unique problem to the continuity of time. The defining features of the growing block's present demand that becoming will occur through the accumulation of instantaneous temporal slices, which inevitably conflicts with the doctrine that time is continuous. The A-theoretical commitments of the block on the one hand, and its B-theoretical commitments on the other, rule out any possibility to resolve this tension. It seems that the answer might be that the growing block theory cannot hold on to all of it. Its defenders will have to give up either some of the theory's commitments, or the continuity of time. Of all the options I considered, the growing event theory and the discrete growing

block seem the most promising. The growing events theorists should be able to hold to the growing block's ontological and dynamical theses, but there are still open questions: if they succeed in disposing of the B-theoretical commitments entirely, would it still be a growing block? And could the theory still support an objective present? If the growing events theorists will be forced to keep to the B-commitments, will they still be able to adopt an A-theoretical account for continuity? The discrete growing block, on the other hand, is able to preserve all of the elements that seemed contradictory on a continuous growing block view.

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