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Cantor's Principle of Finitism or the Actualization of Infinite Potentialities

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Abstract

The paper sets forth a critique of Georg Cantor's philosophical justifications for the introduction of transfinite numbers, and thus actual infinities, in his set theory. "Critique" here has a twofold meaning: 1) the investigation concerns the conditions of possibility of set theory and not its strictly mathematical content, and 2) it consists in a *pars destruens* of Cantor's "dogmatic" defense of his opaque mathematical entities. It is based on the examination of the classical definition of set given by Cantor in his 1883 foundational work, and it is developed through the analysis of some crucial conceptual pairs: one and many, completeness and incompleteness, totality and variability, and especially actuality and potentiality.

Key words: Georg Cantor, set theory, one–many problem, actuality, infinity.

1. Introduction

It could be surprising to find the expression "principle of finitism" next to the name of the mathematician who defended the existence of actual infinities against the dominant Aristotelian tradition. At first sight, an opposite principle attributed to Georg Cantor would be more appropriate. However, as we shall see, this principle and set theory are only superficially contradictories; at a careful analysis, Cantor's finitism in relation to sets is the condition of possibility for his mature mathematical theory of the infinite.

The expression is due to Michael Hallett, as he explains in his *Cantorian Set Theory and Limitation of Size*:

> First, it expresses a certain "finitistic" attitude to sets (mathematical objects) and which is what gives the theory its unity. Namely, sets are treated as simple objects, regardless of whether they are finite or infinite. Secondly, all sets have the same basic properties as finite sets. (1984: 32)

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Thus, the quid of the principle is that it provides the *homogenization* of two different mathematical realms, the finite and the infinite. It is the case because sets have the special property of being treated as single objects, the multiplicity of their elements notwithstanding. Once numbers are defined in terms of sets, this special property is what allows the pivotal passage to a uniform mathematical theory, regardless of the finite or infinite size of its objects.[1]

From a philosophical perspective, the principle of finitism plays an essential role in Cantor's defense of the existence of the actual infinite. According to him, yet, there are two types of infinity. In the 1883 foundational work, *Grundlagen einer allgemeinen Mannigfaltigkeitslehere*, he distinguished within the realm of infinity between an increasable (thus "numerical"), but not merely potential, infinite or *transfinite*, and an unincreasable one or *Absolute*. In this way, he showed how it was possible to treat the former in a rigorous mathematical manner (i.e., as a set), preserving at the same time the human inaccessibility of the latter, identified with God, as it was prescribed by his fervid religious faith. Take the following passages contained in letters written in 1886:

> [The transfinite] is in itself constant, and larger than any finite, but nevertheless unrestricted, increasable, and in this respect bounded. Such an infinite is in its way just as capable of being grasped by our restricted understanding as is the finite in its way.

> [The Absolute] cannot in any way be added to or diminished, and it is therefore to be looked upon quantitively as an absolute maximum. In a certain sense, it transcends the human power of comprehension, and in particular is beyond mathematical determination. (quoted in Hallett, 1984: 13–4)

The emphasis on the capacity for "our restricted understanding" to reach or not such infinities is crucial because of Cantor's conception of mathematical existence. On his view, it is sufficient to give a consistent definition of

1. This is not completely true because of the discovery of the famous antinomies stemming from the "principle of comprehension" that allows the positing of sets that are "too large" to exist. It is quite clear that Cantor never endorsed such a principle *simpliciter* — according to Hallett, the so–called "naïve set theory" is for the most part an illegitimate attribution to Cantor, see (1984: 38). Nonetheless, some of these paradoxical consequences were known by Cantor himself, whose theological response is related, as we shall see, to what he calls "the Absolute." He considered "the Absolute," at least in the last part of his life, appropriately represented by "inconsistent multiplicities" (i.e., multiplicities that are not *sets*), like the multiplicity formed by all sets. Through a different theoretical move, in the successive axiomatization of set theory, these problems have been overridden — or better: dismissed — by the so–called "separation principle." Note, however, that Russell–type paradoxes are based on the same essential feature of sets that we have just sketched: only insofar as they can be members of another set (including itself) one can create new problematic objects such as the "set of all sets" or the "set of all sets that are not members of themselves" and so forth.

a certain mathematical concept in order to guarantee its objective existence. In *Grundlagen*'s vocabulary, we can say that the "intrasubjective existence," that is, the mutual coherence of non–contradictory concepts, implies *ipso facto* their "transsubjective existence," at least in God's mind.[2] Concerning this Leibnizian principle, ultimately based on a theological premise, the real issue at stake is, once again, the possibility of a consistent definition of transfinite numbers in order to infer their real existence. Independently of the plausibility of Cantor's ideas on such a condition, our question is precisely *how* he could justify a "determinate" characterization for "objects" belonging to the realm of the actual infinite. A mathematical determination, in fact, must define them by a reciprocal limitation.[3] In a word, our question is about the way in which he could make definite the infinite, that is, on which basis he could transform the infinite into a unity. In doing so, he maintained aside the Absolute infinite inasmuch it "can only be recognized, never known, not even approximately" (Cantor, 1976: 94). This important aspect of his thought represents a clear heritage of a philosophical tradition that is not limited to Christian authors.[4]

The discussion in the next pages aims at interpreting, following Hallett's subtle analysis, the notion of set and its supposed explanatory role in mathematics. It will be shown why it embodies the philosophical key for the justification of the introduction of determinable infinite mathematical object. In other words, this notion functions as the pre–mathematical condition of possibility for the mathematical results of set theory. In this sense, the paper has a twofold *critical* purpose: on the one hand, it does not concern set theory *per se*, but rather the Kantian issue about *how* it is possible; on the other hand, it examines the conceptual difficulties implied by this philosophical ground, namely the ambiguity of considering a set as a new object (*Ding für sich*), i.e. the limits of what we have called the "principle of finitism."

2. On this Cantorian conception of mathematics as a "free conceptual construction," see Hallett (1984: 13–24).

3. Traditionally, from this observation philosophers have denied both the epistemological access. Even though Cantor accepts the premise, he rejects the conclusion, as he comments Spinoza's famous *dictum* in the *Grundlagen*: "for my mind the proposition *omnis determinatio est negatio* is unquestionably true. [. . .]. What I maintain and believe I have proved in this paper as well as in earlier attempts is that after the finite there is a *transifinitum* (which could also be called *suprafinitum*), i.e. an unlimited gradation of definite modes which in their nature are not finite but infinite, yet which, just as the finite, can be determined by definite, well–defined and mutually distinguishable *numbers*" (1976: 76).

4. Hauser (2013) suggests plausible influences on Cantor's thought by Plotinus and Bruno, among others. Moreover, Cantor was a passionate reader of Spinoza, see Newstead (2011).

2. The definition of set

In the *Grundlagen*, Cantor defines the concept of set in this way:

> By a "manifold" [*Mannigfaltigkeit*] or "set" [*Menge*] I understand in general any many [*Viele*] which can be thought as one [*Eines*], that is every aggregate [*Inbegriff*] of definite elements which can be united to a whole through a law. By this I believe I have defined something related to the Platonic εἶδος or ἰδέα. (1976: 93)

Despite the complexity of the terminology, we can divide the definition into two parts:

a) A set is a multiplicity of definite elements that *can* be united to form a whole.
b) This whole emerges *realiter* as a unit by means of an act of thought that grasps the multiplicity of elements as a *totality* through the acknowledgment of a law.

Cantor points out an objective aspect of sets, i.e., the fact that they are mere aggregates which nonetheless have the power of being a unity, and a subjective aspect, i.e., the actualizing activity of the mind that holds their elements together.

Despite the appearances, the definition implies neither an intentional nor an extensional conception of sets. This amounts to saying that sets are neither entirely determined by some conceptual *notae*, nor by the mere sum of the objects they contain. Notwithstanding the allusion to Plato's notions of *eidos* and *idea*, a set is formed by its elements, whereas, e.g., the idea of man, for Plato, is not composed by individual men. As Hallett says, Cantor "speaks of a set as something 'thought as a thing for itself [*Ding für sich*]' yet '*consisting of* clearly differentiated concrete things or abstract concepts'" (1984: 34). However, we cannot follow him when he claims that since Cantor does not defend an intentional account *tout court*, he accepts an extensional one as a result. Denying the intentional character means to undermine the subjective side of the definition, as Hallett himself confesses: "[the unity of a collection] is something that we as thinking subjects impose on the collection, we 'create' the set (unity) from the elements" (34). According to our analysis of Cantor's text, this is not allowed: even though a set consists of its elements, it is not exhausted by their arithmetical sum — a set is at the same time something more, a new unity.[5]

5. Gianni Rigamonti observes: "un insieme è una pluralità–unità, una molteplicità di oggetti che è sua volta *un* oggetto, altrettanto determinato che i suoi elementi. *Ma questo non è un concetto estensionale*. Qui l'accento non cade sulla pura e semplice estensione ma nel suo darsi come un *nuovo* oggetto" (1992: xxxiv–xxxv).

We can make this point clearer if we consider that in the *Grundlagen* Cantor's main problem was to make acceptable the existence and the mathematical studying of actual infinities. The typical example employed was the sequence of natural numbers, which functions now for us as a paradigmatic case to verify our interpretation. The following passage is taken from a letter written in 1887 (quoted in Hallett, 1984: 33):

> The first, simple fact, accessible to everyone, on which the theory of the transfinite is based is the simultaneous boundlessness and yet the definiteness in itself of the series of all finite cardinal numbers 1, 2, 3, ..., *n*... viewed as a constant set of clearly differentiated things.

The text is once again divided into two parts. Firstly, we are told what the aggregate of finite natural numbers *objectively* is, i.e. an unlimited series of definite elements. Thus, it is a potential, variable, incomplete process of greater and greater numbers. Secondly, it can be *viewed*, though, as an actual, constant and complete totality from a peculiar *subjective* perspective. In this way, Cantor bridges the intentional and extensional nature of sets. Sets *consist of* nothing but their elements since objects are their "substance" (taken for granted that they exist, in the mathematical realm or somewhere else). At the same time, a set is not merely the sum of its elements, i.e. an aggregate, but, in addition, a *mode* of seeing them. The adjunction is not arithmetical, *objective* (we do not add *another* element to the series), but *subjective*, we consider the series under a different light through the law of succession. As a consequence, the multiplicity becomes a unity *for the subject*. In a nutshell, the series as a whole becomes mathematically determinable, actual, one — which means the same thing for Cantor.[6]

At this point, we can ask two related questions corresponding to the two sides of this definition of set:

a) How *can* an infinite multiplicity of objects form a unity? What does potentiality mean in this context?
b) *For which subject* such a multiplicity becomes a unity?

3. What does potentiality mean?

In our reading, the givenness of mathematically defined objects represents the *matter*, as it were, from which the subject draws a *law* to transform the aggregate into a set. As it is so far clear enough, for Cantor not every aggregate can form a set. Thus, there must be something in the objective

6. This interpretation is inspired by Chiurazzi's reflections on the nature of modality in (2002).

series of objects, e.g. in the series of natural numbers, that "suggests" the active recognition of a kind of unity and consequently a new actuality. In fact, a completely random series of objects cannot in any case point to a limit, so that the subject has no chance to close that multiplicity in a totality. Metaphorically speaking, in this latter circumstance, the matter is entirely passive. On the contrary, to justify the case in which the disposition of objects embodies some potentiality and permits its enclosure by a subjective sight, we can proceed *by analogy* with other less controversial mathematical cases.

In particular, we have to consider the real mathematical keystone in the theory of transfinite numbers, that is the second generating principle introduced in the *Grundlagen*: "if any definite succession of defined whole real numbers is given of which there is no greatest, then on the basis of this second principle of generation a new number is created, which can be thought of as a *limit* of those numbers, i.e. can be defined as the next greater number to all of them" (1976: 87). This principle is nothing but the mathematical realization of the philosophical principle of finitism. In fact, if a certain given aggregate is to be conceived of as a set, then it must be determined by a limit.[7] Thus, "the second generating principle facilitates the transition from a given 'definite succession' of numbers to its least upper bound. Definiteness here means that the numbers appearing in the succession form a set. Since a set is a finished thing for itself, each definite succession must have a least upper bound" (Hauser, 2013: 173). So that, instead of considering that series pointing to an unspecific infinity, he regarded it as directed to a definite limit, yet besides the domain of natural numbers:

$$1, 2, 3, \dots n \dots \omega$$

This idea is not completely new, since it can be thought of as an extension of an analogous process of generating and identifying numbers through a succession. For instance, the convergent succession

$$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16} \dots$$

has the number 1 as limit since it is *greater* than every number in the succession and at the same time it is the *smallest* one of every number

7. As Dauben claims: "[Cantor] was primarily interested in sets as a whole because only in such terms could the transfinite numbers be defined. If one did not regard the set of integers $\mathbb{N}$: 1, 2, 3 . . . as conceivable as an entity, as a completed set, then there was no way to produce even the first transfinite number" (1979: 170).

greater than them. With respect to the sequence of positive integers, ω instantiates the same properties.

More suitably, a definite succession of rational numbers can be elaborated to define even an irrational number:

3 3,1 3,14 3,141 ...

If we call π the limit of this succession, then we are establishing a number of a different nature from those that are the elements of the succession. In the same way, according to the second principle of generation, the definite succession of natural numbers is the necessary and sufficient condition that allows the passage to the limit and the creation of a number of a new kind.

Historically, Cantor's principle of finitism stemmed from his attempt of definition of real numbers in terms of "fundamental sequence," i.e. a succession of rational numbers, similarly to the one we have just sketched. Already in this 1872 work, Cantor faced the opacities that we are trying to elucidate. In this occasion, he interpreted for the first time these definitions in the finitistic manner that then he would have adopted for transfinite numbers. Namely, "real numbers defined by the methods of 1872 are certainly taken as single objects (individuals)" (Hallett, 1984: 31). Granted that they *are* those sequences, they gain nonetheless a new status equal to that of rational numbers:

> they are either themselves collections or are co–ordinated with collections or a domain of collections, and these themselves therefore should be subject of predication, be the arguments of functions, etc. But this means accepting either way that collections (multiplicities) can also at the same time be single objects (individuals). (31)

This passage is again made possible by the understanding of infinite sequences as completed sets through the position of a limit.

Even though the problem of the principle of finitism is already present in the definition of real numbers, this process of generation appears undoubtedly less debatable: we are more inclined in accepting that irrational numbers, despite their definition in terms of successions, really exist as individuals, just like natural numbers do. That is the case because in the successions we mentioned, on Cantor's view, we feel that the tension to a definite limit is somehow grounded in the disposition of the elements themselves. There is objectively a tendency that indicates an ultimate point of destination. But this means that the infinite sequence is in some sense limited. According to Cantor, this inner inclination is sufficient to allow the subjective passage to actual infinity, considering the succession as a completed domain with a precise upper limit that is "external" to the series. The decisive point here is the acknowledgement of the fact that real numbers, defined through Cantor's method, are already a case of an *infinitum in actu*. As he claims in the *Mitteilungen*:

> The transfinite numbers are in a certain sense themselves *new irrationalities* and in fact in my opinion the best method of defining the *finite* irrational numbers is wholly similar to, and I might even say in principle the same as, my method described above of introducing transfinite numbers. One can say unconditionally: the transfinite numbers *stand* or *fall* with the finite irrational numbers; they are like each other in their innermost being; for the former like the latter are definite forms or modifications of the actual infinite. (1887: 99)

In truth, there is a major difference: fundamental sequences are convergent whereas the succession of natural numbers is not. However, through the second principle of generation, Cantor recognizes an analogous internal necessity in $\mathbb{N}$ towards a limit and postulates ω as this new irrationality or new actual infinite number.[8] As in the previous case about real numbers, the twofold process of generation starts with the first objective potentiality "internal" to the succession (what in the *Grundlagen* Cantor calls the improper–infinite) and the second subjective actual position of a limit, "external" to the succession (proper–infinite). An external subject is necessary for the conversion of the succession in a set since the potentiality *in* the succession cannot succeed in ending its course. Therefore, the generating passage to the limit is *immediate*, it is a kind of modalization of the sequence and not the final stage of a process. It arises from a different *way* of treating the very same objects that are not annihilated but *comprehended* as non–variable or completed. Following Cantor's theological images, we could say that this transformation is like a religious revelation. Cantor sees a limit where the others have seen nothing but the indefinite continuation of natural numbers. We must then analyze which subject is able to bring about this type of comprehension, i.e. gathering the elements of an infinite potential sequence as an actual *Ding für sich*.

4. Which is the actualizing subject?

What kind of subject can operate such an imposition on this numerical matter? To explain again our line of interpretation, this "imposition" does not involve an affection *of* the succession in itself, rather a different perspec-

8. Concerning the analogy we have underlined, Rigamonti adds: "è vero che c'è una differenza importante: nel caso dei numeri irrazionali la successione generatrice è convergente, nel caso dei transfiniti non lo è. Ma le differenze non devono nasconderci i caratteri comuni: Cantor non fa che estrapolare, lasciando cadere la condizione di convergenza, un processo di creazione di numeri di tipo nuovo che a tale condizione, inizialmente, era legato" (1992: xxviii).

tive *on* it. In Cantor's texts, two possible actors are hypothesized for the "actualization of an infinite potentiality": a psychological and a theological subject. Both options imply some insuperable difficulties.

For the psychological case, objections can be raised even if we put aside the circumstance of an infinite collection. Since the extensional aspect cannot be removed, a human mind should be able to make a unity out of something that is *de facto* a multiplicity. As Hallett notes: "two apples on my table remain two apples no matter how I try to conceive of them as 'one thing'" (1984: 35). Similarly, in which sense can one consider, say, the collection of the first three natural number as *one* thing? Unless we get rid of the extensional character of sets, no psychological effort can transform a collection into a unity. *A fortiori*, the same goes for infinite collections. Cantor often associates the properties of being total, complete, constant, actual to sets. But "we are incapable of directly perceiving or intuiting the whole of an infinite domain either as a totality or indeed one by one" (26). In other words, the psychological subject is not able to comprehend an infinite sequence as a set.

Apparently, for a theological subject this limitation could be overcome. Hallett remarks: "there is evidence to suggest that for Cantor much more important than our ability of conceive of a collection as 'one' was God's ability to do so, and this of course fits well with his theological justification of his ontology" (35). Cantor quotes an entire chapter of Saint Augustine's *De Civitate Dei* in the *Mitteilungen*, suggesting that this idea was already present in the philosophy of a Christian authority. The twofold rhetorical aim was to prove the existence of an admired alleged predecessor of his theory and to protect himself from possible accuses of heresy by Catholic theologians. The central sentences are these:

> Every number is defined by its own unique character, so that no number is equal to any other. They are all unequal to one another and different, and the individual numbers are finite but as a class they are infinite. Does that mean that God does not know all numbers, because of their infinity? Does God's knowledge extend as far as a certain sum, and end there? No one could be insane enough to say that. [...]. Although the infinite series of numbers cannot be numbered, this infinity of numbers is not outside the comprehension of him whose understanding cannot be numbered. And so, if what is comprehended in knowledge is bounded within the embrace of that knowledge, and thus is finite, it must follow that every infinity is, in a way we cannot express, made finite to God, because it cannot be beyond the embrace of his knowledge.[9]

Two aspects are crucial here: on the one hand, the fact that "infinity is made finite to God," i.e. it is closed within its *knowledge* as one thing (it is

9. Augustine, *De Civitate Dei*, book XII, chapter 19, quoted in Hallett (1984: 35–36).

the subjective intervention that bounds an objective infinity), on the other this passage is not understandable for us, it is produced "in a way we cannot express." The totality of numbers is gathered in God's mind as a completed domain, as he contemplates it as a single object.

The direct consequence of this conception is that studying set theory we do not know what we are doing. Mathematics would be a sort of divine practice whose accessibility pertains exclusively to God. At most, we would employ symbols having objective denotata, but they would be completely mysterious for us. Our belief about the meaningfulness of set theory should be based on the previous theological argument but it is easy to notice that the hypothesis seems *ad hoc*. "Indeed, it seems that God is brought in essentially to bridge a gap (between a collection and its unity as a set) that we ourselves cannot bridge" (Hallett, 1984: 37). Without any doubts, this weak assumption is grounded in Cantor's entire philosophical framework and undermines even his mathematical results.

But there is a more radical objection whose force is independent of the introduction of some theological elements. It relies on Cantor's late distinction between consistent and inconsistent sets that he explained at first to Dedekind in an 1899 letter:

> a multiplicity can be such that the assumption that *all* its elements "are together" leads to a contradiction, so that it is impossible to conceive of the multiplicity as unity, as "one completed thing." Such multiplicities I call *absolutely infinite* or *inconsistent multiplicities*. [...]. If on the other hand the totality of the elements of a multiplicity can be thought of without contradiction as "being together," so that they can be gathered together into "one thing," I call it a *consistent multiplicity* or a "set." (Quoted in Jané, 1995: 375)

We are not mainly interested in the reasonability of this dichotomy, but rather in its relationship with the problem we met once Cantor introduces the theological subject to explain the formation of sets. This conceptual opposition should avoid the difficulties of the first period of set theory exposed by self–reference paradoxes. According to Cantor, these apparent contradictions arise from considering some inconsistent multiplicities, i.e. collections which cannot be completed, actual etc., as sets. In summary, whatever collection that leads to a contradiction cannot be thought as a thing in itself, since the passage from potentiality to actuality is thus prevented. Since Cantor accepts the idea that this type of multiplicities represents in an appropriate way the Absolute, then it is clear that he does not deny the existence of those "inconsistent multiplicities"[10], but he

10. On the contrary, the consequence of Zermelo's axiom of separation accepted in post–Cantorian set theory rules out exactly the possibility of the existence of those "multiplicities."

conceives of them in some sense "veiled" by the paradox. To understand what he means, we take, for example, the universal set as a collection of that kind. Suppose that the collection of all things forms a set, **u**, whose cardinality is C. By definition of universal set, the power set (the set of all sets) of **u** must be a member of **u**, so that its cardinality C: must stand in this relation: $C1 \leq C$. However, for Cantor's theorem, given a set, the cardinality of its power set is strictly greater than its own cardinality, thus in this case: $C1 > C$. That is a *reductio ad absurdum* of the first premise: the collection of all things cannot form a set.

Although Hallett underlines the vagueness of the distinction, even only the case of an inconsistent multiplicity just mentioned is sufficient to refute the hypothesis of God as a possible "collecting subject." This is the case because there is no reason according to which these collections should be fallen beyond God's knowledge. It would mean that God's science would extend only up to a certain set ignoring the larger ones. Again, Augustine would affirm that "no one will be insane enough to say that." Thus, even the universal set should be made finite to God. But that is contradictory and even God cannot override contradiction. The problem here is that the argument is supposed to "legitimize ω but [it] falls short of the mark because (if it proves anything) it proves too much" (Jané, 1995: 399). In fact, if it works for the collection of natural numbers, it must work for every other infinite collection. This observation reveals that Cantor's reference to God is irrelevant and that, as Jané notes, "the trouble with this [Augustinian] argument is not its appealing to God, but its taking the absolute as actual, as one thing" (399). Thus, we are put back to our first step: the oneness of sets remains inexplicable because even God's hypothesis cannot justify when a collection can or cannot become a set.

If it is so, it seems to be no help to appeal to a transcendental subject as Gödel pointed out in *What is Cantor's continuum problem?*:

> Note that there is a close relationship between the concept of set [...] and the categories of pure understanding in Kant's sense. Namely the function of both is "synthesis," i.e. the generating of unities out of manifolds (e.g., in Kant, of the idea of *one* object out of its various aspects). (Quoted in Hallett, 1984: 302)

Those problems we encountered before are simply replied in the new context. Even worse, a supposed synthesis of a transcendental subject should rely on time as in the Kantian doctrine of transcendental schemas. But throughout our analysis we have emphasized the narrow detachment between a variable and never–ending potential infinite and a constant and actual infinite, whose comprehension breaks any temporal construction.[11]

11. Cantor says, for instance: "My conceptual grasp of the transfinite excludes properly and from

Even though it could be a valid theoretical path to give a satisfactory account of the unity of multiplicities, certainly it has not been followed by Cantor. God's faculty to keep together infinite collections was intended to clarify precisely this mysterious passage from finitude and process to an actual non–temporal infinity. But as we have seen, that is not convincing.

Looking at the development of set theory, Hallett is drastic: "we must give up the idea that our use of sets is anything like a Kantian 'synthesis of manifolds into unities.' And it is precisely because set theory took this strong turn away from 'constructive explanation.'" Moreover, "axiomatization went hand in hand with the divorce from any attempt to understand what sets are or what conceptual role they play. We cannot say with any kind of conviction what sort of things sets are, so we attempt a type of ostensive definition of them through axiomatization or 'listing'" (303). In this interpretation, post–Cantorian axiomatization appears as a pragmatic enterprise that is the coherent heritage of the failure in elucidating the notion of set during the first period of set theory. The empirical fact that this theory is useful as the ultimate framework for every other mathematical discipline does not prove anything since "one is still left with the conceptual mystery of why all mathematical objects *should* be sets ('unities out of manifolds'). And in any case conceptual problems do not go away just because the theory they concern is successful" (305). In this sense, Cantor's struggle with the foundation of his doctrine represents a chapter in the history of the ancient problem of "one and many," whose unsatisfactory solution, on Hallett's view, occasioned, exactly because of its vagueness, the more recent axiomatic developments.[12]

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the beginning 'process,' since this denotes a 'change.' [. . .] a transfinite process properly speaking seems impossible to my understanding of the 'transfinite,' because here the two mutually exclusive predicates 'definite=constant' and 'variable' would be joined" (quoted in Hallett, 1984: 28).

12. Whether the unity out of an infinite multiplicity is occasioned by a *symbolic* subject, instead, is not even taken into consideration. The hypothesis set forth by Jacob Klein in a lecture delivered in 1932 (published in 1985 in a collection of essays) induces to regard the origin of this magical unity in the nature of mathematical *symbols* themselves. The "mythological" justifications for the employment of symbols supposedly denoting the actual infinite occur when the logic of symbolic mathematics is not only blindly practiced but, at the end, misunderstood. Instead of grounding *externally* the signification of symbols, the meaning is to be understood starting from the rules of the calculus itself. But this is another story. See on this point Stenlund (2015: 34–42).

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