– ISBN 978-88-255-2179-5
DOI 10.4399/978882552179511 – pag. 169-190 (dicembre 2018)

Exit from the self–incurred contradiction

On Kant's mathematical antinomy and its consequences

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Abstract: The paper considers Kant's presentation of the antimony as an indirect proof of transcendental idealism. By examining the two mathematical antinomic conflicts, it purports to show that Kant's proofs of the respective theses and antitheses turn out to be inconclusive for the alleged dogmatist. Not only does the latter not fall into an inevitable contradiction, he is also able to solve it in advance, as illustrated by Leibniz's theory of the creation of the world and Kant's pre–critical physical monadology. Since the source of the antinomy, it is claimed, lies rather in the conflicting requirements for the possibility of experience as exposed by Kant, it is the transcendental idealist who is haunted by an inherent contradiction. This has, it is argued, important consequences for Kant's system, as it invalidates the principle of complete determination and in general leads to a peculiar top–down gappy ontology.

Keywords: transcendental idealism, transcendental dialectics, antinomy, physical monadology, complete determination.

The antinomy occupies a privileged place in the genesis and structure of the *Critique of Pure Reason*. From the very beginning of his academic career, Kant displayed a marked conceptual interest in the phenomenon of error.[1] According to one of the two canonic accounts given by him in a late letter to Garve, it was precisely this "scandal of ostensible contradiction of reason with itself" that first aroused him from his dogmatic slumber.[2] Similarly,

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1. See, for instance, R 3706 (1760–64?; AA 17: 242): "This does not mean to philosophize, if one solely seeks to establish that something is a delusion, a deception of the understanding, but rather one must also learn to have insight into how such a deception would be possible." Kant's writings will be cited by volume and page number of the reference edition, the so–called *Akademie–Ausgabe* (AA; see Kant 1900–), save for the *Critique of Pure Reason* (*KrV*), which will be cited according to the second (B) and first editions (A). The so–called *Reflexionen* will be referred to as R followed by a number and the approximate dating according to Adickes. The translations, where available, are taken from the *Cambridge Edition*.

2. Letter to Garve, 1798, September 21, 1798 (AA 12: 257–258). See also the Letter from Hartmann, September 1774 (AA 10: 169); R 4275 (1770–71; AA 17: 492); and, in relation to the breakthrough of '69, R 5037 (1774–76; AA 18: 69).

when Mendelssohn called him "alles zermalmender Kant," or when Hegel deplored the disappearance of metaphysics, they both essentially referred to Kant's treatment of antinomy. The effect in question was largely brought about due to the skillful deployment of theatrical dramaturgy within a theoretical argument. For what in Kant's view represents a general consequence of flying beyond the boundaries of possible experience — a variety of metaphysical schools fiercely fighting one another without ever being able to obtain a decisive victory — was portrayed in the Critique as a natural, in a sense even necessary product of human cognitive machinery. Moreover, this "strange phenomenon" offered Kant an opportunity to formulate an "indirect proof" of his system (see *KrV*, B534/A506). For if he can show that reason in the end inevitably contradicts itself unless we subscribe to the standpoint of transcendental idealism, while from his standpoint the contradiction disappears, then transcendental idealism is not only valid, it is the only valid philosophical system.

In our paper, we are going to assume this point of view to investigate Kant's treatment of the mathematical antinomies. In a sense, we propose to employ the skeptical method used by Kant in the antinomy chapter and apply it to his very treatment of them. In this way, we would like to scrutinize the validity of Kant's indirect proof and examine the real consequences of the antinomy for both conflicting doctrines.

But before we start, we have to remind ourselves that in order to be conclusive, Kant's indirect argument must meet at least two requirements. First, the opposition between the two doctrines has to be exhaustive and well–defined. Kant often implies that a dogmatist is a transcendental realist who does not admit of the ideality of space and time, taking appearances to be things in themselves. For our purpose, however, we are going to understand dogmatism in a negative reference to the highest principle of all synthetic judgments, saying that "the conditions of the *possibility of experience* in general are at the same time conditions of *possibility of the objects of experience*" (*KrV*, B197/A158). As this definition singles out Kant's standpoint against all previous philosophical systems, it seems to comply better with his self–understanding. It is important to note, though, that according to this reading, the mere ideality of space and time does not yet suffice for something to pertain to transcendental idealism. What is decisive is rather the question whether the object under consideration is, or is not, determined the way it is independently of any cognition. In this sense, for instance, Leibniz's conception of space is still dogmatic: although space is a mere epiphenomenon of relations that hold among the real, essentially non–spatial substances, for Leibniz, this structure is completely explainable in objective terms, without any reference to the subject of cognition.

And second, the argument has to be compelling for the dogmatist. In the antinomy chapter, Kant is not addressing the transcendental idealist, who has presumably already been convinced, he is talking to dogmatists. As a consequence, Kant insisted that in conceiving his proofs he had "not sought semblances in order to present . . . a lawyer's proof"; on the contrary, each of these proofs was said to be "drawn from the nature of the case" (*KrV*, B 458/A430). He went even further in the *Prolegomena,* where he solemnly declared that he would personally "vouch for the correctness of all these proofs" (AA 4: 340), and even incited the reader to find eventual fault in these, as he called them, "equally evident, clear, and incontestable proofs."[3] This discursive strategy is sound, as it were. For if his proofs turned out to be deficient, if a contradiction happened to arise from some neglect in the use of reason, he would be unable to bring his indirect point home.[4] Again, Kant is supposed to convince the dogmatist, operating on her terrain and using her assumptions.

This is how we propose to read Kant's presentation of the mathematical antinomies. In the first step we are therefore going to look into what the antinomic dispute between exemplar dogmatists looks like according to Kant, and then examine the consequences thereof for transcendental idealism. We are going to play Kant's game, but not with Kant's intentions. We are going to play by his rules to see if Kant himself complied with them. In the end, these rules boil down to one simple prescription: It is not allowed to presuppose the validity of transcendental idealism. To play Kant's game is, in brief, to play by the rules of the dogmatist.

1. The first antinomy

The first antinomic conflict deals with the world (i.e., the whole that is not itself a part) as an extensive whole; more precisely, it refers to the question whether the material world is limited in space and time or not, whether it is finite or infinite. The thesis reads as follows:

> The world has a beginning in time, and in space it is also enclosed in boundaries. (*KrV*, B454/A426)

The thesis consists of two parts, treating space and time separately. The same duality is then also reflected in Kant's formulation of proofs, as the argument for spatial boundedness relies entirely on the boundedness of events in

3. See *Prolegomena* (AA 4: 341): "I therefore desire the critical reader concern himself mainly with this antinomy . . . I promise to answer for each proof I have given of both thesis and antithesis."

4. See *KrV*, B512/A484: "Thus the dogmatic solution is not merely uncertain, but impossible."

time, which has allegedly been proved already. By adopting such a procedure, one might observe that the proof was "infected with transcendental philosophy" from the very beginning. Be that as it may, we are going to limit our consideration to the spatial part of the thesis. The relevant proof reads:

> For if one assumes that the world has no beginning in time, then up to every given in time an eternity has elapsed, and hence an infinite series of states of things in the world, each following another, has passed away. But now the infinity of a series consists precisely in the fact that it can never be completed through a successive synthesis. Therefore an infinitely elapsed world–series is impossible, so a beginning of the world is a necessary condition of its existence. (*KrV*, B 454/A426)

The argument is dogmatic indeed. It is even traditional, since it was invented back in the antiquity, and was later used on a regular basis in medieval disputes under the heading *de aeternitate mundi*. Its thrust is that one cannot produce infinity by successive synthesis, in the mode of counting, while by affirming that the world is eternal we say precisely this. To illustrate the point, let us consider counting the Earth's revolutions around the Sun (or, rather, the Sun's revolutions around the Earth); if the world was eternal, then by now we would have come to an infinite number, for in the presupposed infinite past the Earth would have already made infinitely many revolutions.

This is at least how the traditional argument goes, and Kant is here content to simply repeat it. What should surprise us, however, is that this old problem has long had its more or less generally recognized solution.[5] It is true that in the event of the eternity of the world we should by now have counted to infinity; this is, let us assume, contradictory. However, the problem lies not in eternity, but in the very idea of *counting*. This idea is inappropriate insofar as it assumes that somewhere, at a certain point in the past, we *began* to count. It therefore assumes that a certain revolution was *first*; it assumes that at some point the Earth *began* to revolve; it presupposes, in short, that the world has a beginning in time. But since this is exactly what we wanted to prove, the argument is circular.[6] Without this arbitrary assumption, the only thing Kant could prove with this argument is that if the world has no beginning in time, then it has always already existed for an infinite time, as was noted for example by Russell (see 2009: 126–127), to name but one. No matter how far back into the past we go, we will inevitably find that for every finite past there is *always an infinite past lying before it*. And this is definitely no contradiction.

5. For a short presentation of some major positions in the dispute, for instance those of Philoponus or St Thomas, see Wood (2010).

6. This is also Hegel's objection in the *Science of Logic*; see GW 21: 229. Interestingly, Hegel posits the presupposed limit not at the beginning of time, but rather at its end, at the *present* time. "But," Hegel adds, "this difference is inessential."

But if, from the dogmatist's standpoint, Kant's argument is evidently invalid, we have to ask ourselves what made Kant formulate it in the first place. How was *his* error possible? The reason is, perhaps, that the eternity of the world relates not only to the problem of infinity but also to that of the whole. The concept of the world is the concept of the whole. It is well known, however, that for Kant there were two types of the whole: *totum analitycum* and *totum syntheticum*, or *totum* and *compositum*. In the first case, the whole is a condition of its parts, which can exist only within it; in the second, the whole is constituted through a gradual aggregation of pre–existing parts[7]. If, therefore, in the analytic whole the idea of infinity is readily acceptable since its parts are given together with it regardless of whether their number is finite or infinite, the synthetic whole, on the contrary, cannot be infinite because infinity cannot be produced through a gradual synthesis.

The question is, then, why Kant treats the temporal extension of the world as a synthetic whole. For Kant, space and time are rather all–encompassing *analytic* wholes, and in the Transcendental Aesthetic he accordingly explicitly described them as infinite.[8] In the Transcendental Analytic, however, Kant presented it as an axiom of intuition that all appearances, that is, all materially occupied parts of this infinitely extended intuitive space, are for their part extensive magnitudes generated through a successive synthesis of their parts.[9] For Kant, every cognized object in space and — because of the conceptual parallelism — time consequently constitutes a synthetic whole.

Thus we can imagine that Kant considered the world as a synthetic whole on the basis of the requirements developed in the Transcendental Analytic. Indeed, it turns out that *for Kant* the idea of an infinite past series is impossible, since according to *his view* time in a sense runs *backwards*. To see why this is so, it suffices to take a brief look at Kant's treatment of actuality, or *Wirklichkeit*. Although it does imply a reference to perception, a thing does not need to be perceived directly in order to be considered actual. Following the second postulate of empirical thought, Kant declares that "everything is actual that stands in one context with a perception in accordance with the laws of empirical progression" (*KrV*, B521/A493). This subtle distinction proves to be relevant for those things in particular that, due to the specific constitution of our senses, cannot be objects of immediate experience, such as "magnetic matter" (*KrV*, B273/A226), things that are simply located too far from us to be perceived immediately, such as the "inhabitants of the

7. See, for instance, R 3789 (1764–66; AA 17: 293). Kant adds: "*Spatium* and *tempus* are *tota analytica*, bodies *synthetica*".

8. "Space is represented as an infinite *given* magnitude … (for all the parts of space, even to infinity, are simultaneous)." (*KrV*, B39–40)

9. "Every appearance as intuition is an extensive magnitude, as it can be only cognized trough a successive synthesis (from part to part) in apprehension." (*KrV*, B203/A163)

moon" (*KrV*, B521/A492), and finally, and most interestingly, "the real things in the past" (*KrV*, B523/A495), so dear to Meillassoux, which existed in a past perhaps so remote that no finite rational subject could have possibly observed them directly. In this respect, Kant claims that those events are nonetheless actual if only I can "represent to myself that, in accordance with empirical laws, or in other words, the course of the world, a regressive series of possible perceptions… leads to a time–series that has elapsed" (ibid.). That is to say, for Kant, the past can be perfectly *actual as a reconstructed past*, a past that in a sense runs backwards, "starting with the present perception, upward to the conditions that determine it in time" (ibid.). However, there is a limit to the existence of such a past: since the reconstructive synthesis can never reach to infinity, no infinite past is possible. In Kant, therefore, there is no infinitely remote past.

It turns out that for Kant the proof of the thesis is thus conclusive after all. But this only makes it clear that, in contravention of the rules of the game, it holds *only* on condition that one accepts transcendental idealism.

The relevant antithesis reads: «The world has no beginning and no bounds in space, but is infinite with regard to both time and space». (*KrV*, B455/A427). In accordance with the apagogical method, the proof of the temporal part starts by assuming the opposite, that is, that the world did begin at a certain moment in time. The argument rests on the contention that, following the definition of "beginning," this would imply the existence of an empty time before the beginning of the world. Surprisingly enough, Kant does not seem to object to it, he merely adds that in this event the world would not have emerged at all. In such an empty time, all temporal parts would be completely alike, none having any specific mark that would distinguish it from the others, which means that no single moment could provide a reason why the world should begin in one specific moment rather than another. It follows that either the world has always already existed or else it could have not begun to exist at all. But the world does exist, and consequently it must have existed infinitely.

Once again, however, this proof does not have the required argumentative force. We may start by objecting that Kant's definition of the beginning as an event that is preceded by a time in which something was not, already seems to preclude the answer. Since the world is the total sum of all things, not just some parts of the world, the real question should rather be whether it is possible to speak of the existence of *time before the beginning of the world* at all. In this respect, Leibniz would simply affirm that the world was created in the first moment of time, that is, that it was only with the existence of the world that time started to exist, too. In this sense, the proof is not apagogical, but simply affirms what it is supposed to prove.[10]

10. This is also Hegel's assessment in the *Science of Logic*; see GW 21: 231.

In addition, one might directly challenge Kant's objection against the possibility of the arising in an empty time. It is true that if we assume the existence of absolute time, no distinctive feature would provide a sufficient reason why the world should begin in one moment rather than another. However, even Kant did not consider to be an analytical truth that every event has a reason or a cause; and besides, a follower of Newton could always claim that the ultimate reason lies in the will of God, who, as a matter of fact and for no reason, decided to create the world in that very moment. Once again, the proof would clearly fail to impress the dogmatist.

It seems, on the other hand, that it is Kant who is obliged to think of a temporal event in a way that would make the beginning of the world contradictory. In effect, in the first analogy of experience, Kant demonstrated that, as a necessary condition of possible experience, the very notion of change presupposes the identity of what changes,[11] and that, consequently, there must be something in the world that persists through all time. This something that persists, usually called substance, thus provides a sufficient proof that *Kant's world* could not have had a beginning. In a similar way, in the second analogy, Kant demonstrated that each event receives its temporal location only in relation to some preceding event, according to the law of causal connection. It is for him that no event becomes part of the world without being determined by its logically and temporally preceding cause. But again, since for Kant this is something that can be established only with transcendental idealism, it is quite clear that it will not convince dogmatists.

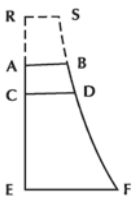
In the end, our brief examination of the first antinomic conflict leads to a result that is in complete disagreement with Kant's initial assurances. The relevant proofs are apagogical in appearance only; they are unable to establish that the dogmatist will inevitably contradict himself, and they are not necessarily conclusive for the dogmatist either. These are strong statements. It is therefore fortunate that we can provide them with additional support from history. Incidentally, Kant's presentation can be read as an *a priori* reconstruction of a confrontation that actually took place between Clarke and Leibniz. In their famous exchange of letters they not only pretended to use the apagogical method, on closer examination it soon becomes evident that they actually argued past each other rather then against each other, reading one's own presuppositions into the other's theory, especially with

11. "Arising and perishing are not alterations of that which arises or perishes. Alteration is a way of existing that succeeds another way of existing of the very same object. Hence everything that is altered is *lasting*, and only its *state changes*." (*KrV*, B 230/A187) See also R LXXVIII (AA 23: 30): "All arising and perishing is only the alteration of that which endures (the substance), and this does not arise and perish (thus the world also does not)."

regard to space and time.[12] In this way they succeeded in showing that under *such* presuppositions, the other's theory involved a contradiction; but as for themselves, both remained perfectly consistent. For instance, being good Christians, they both affirmed that the world began at a certain time: regarding the possibility of an empty time before the creation of the world, which necessarily constituted a "non–entity" for Kant (*KrV*, B461/A433), Clarke simply attached it to God, while Leibniz called it merely imaginary.

It is Leibniz in particular who is of special interest to us. Not only does he prove that the dogmatist's position is free from contradiction, he aptly illustrates how to think thesis and antithesis together without being forced to accept transcendental idealism. Leibniz advocated relational theory, claiming, basically, that space and time are abstract orders arising out of relations sustained by actual simple substances. According to this doctrine, space and time are contingent with respect to both their existence and their characteristic properties. For instance, if the law governing the force of attraction were different, then, perhaps, our space would have more than three dimensions.[13] It has been regularly argued against Leibniz that in this conception there could be no empty space and time, which would necessarily render them materially infinite. The objection is wrong, however (with the exception that it does hold for Kant). While it is true that, in Leibniz, there would be no space without a plurality of substances, the properties of the order of coexistence pertain not only to actually existing substances, but "to possible ones as well, as if they existed." Something similar also holds true for time (though one–dimensionality does complicate the matter). In Paragraph 55 of the Fifth Letter, Leibniz writes:

> But yet absolutely speaking one may conceive that an universe began sooner than it actually did. Let us suppose our universe or any other to be represented by the Figure AF, and that the ordinate AB represents its first state and that the ordinates CD and EF its following states; I say one may conceive that such a world began sooner by conceiving the figure prolonged backwards, and by adding to it SRABS. For thus, things being increased, time will be also increased. (Leibniz & Clarke 2000: 50)



12. For instance, they asked each other whether God could have created the world at some other time or in some other place—say, several feet to the left or right. For a closer examination, see Vailati (1997).

13. Such was at least Kant's conjecture in his early book on the *living forces*; see AA 1: 24.

Although the world was presumably created at a certain point in time, we could, relying on the lawful regularities instantiated in the actually existing world, nevertheless conceive of a *time before* the beginning of the world: to this effect, we would simply need to prolong the past backwards according to the rules of actual temporal succession. This time would turn out to be unreal, merely *imaginary*, because there really was no world and no time before the creation; still, this imaginary past would prove to be completely determined, marked by all the usual properties, and in general allowing us to speak of it in exactly the same way as we speak of the really existing time.

By using this distinction between real and imaginary time, Leibniz would finally bring thesis and antithesis together. He would affirm that, from an absolute point of view, the world is indeed bounded in time, but that beyond this temporal border there is an imaginary time extending into the infinite past. To illustrate his solution, let us suppose—as some calculations made upon the Bible seem to suggest—that the world was actually created around 3500 BC. In this case, there was no time and no world before that date. However, on the basis of the *initial state* of the world (which includes fossils), and on the basis of the presumably *stable order* governing the things of the world *after* 3500 BC, for us, the world would have stretched back infinitely. Researching the history of the natural and the human world, we would thus be right to speak of human evolution before that time, or of the formation of the Solar system. Seen from the outside, the world would then indeed possess its absolute beginning; however, for us, *being inside*, it would be *impossible* to observe that in going back we have, at a certain point, crossed the border between the real and the imaginary, and *entered into the imaginary past*.

For Leibniz, there was obviously no contradiction involved here, quite the contrary. And we can safely assume that Kant knew it. Not only was he familiar with the Leibniz–Clarke controversy,[14] a similar conception was explicitly expounded in the booklet *De aeternitate mundi impossibili*, published by Martin Knutzen, Kant's teacher.

2. The second antinomy

The second antinomic conflict deals with the question whether the composite substance, or matter, is infinitely divisible, that is, whether it ultimately consists of some fundamental parts, which are not capable of further division. The thesis will posit a bottom limit of the division, and therefore the

14. See R 4767 (1775–77; AA 17: 700).

existence of simple substances, while the antithesis will insist on the infinity of the process of division. The thesis, then, reads:

> Every composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed of simples. (*KrV*, B462/A434).

Again, the proof starts with the apagogic assumption that composite substances do not consist of simple parts. To illustrate the absurd consequences such a hypothesis would imply, Kant invites us to perform a thought experiment. If we removed in thought all composition, he claims, then in the absence of simples nothing would be left over. Apparently, Kant is unable to accept such a consequence, presumably since we started with something substantial. Therefore, he argues further, it is either impossible to remove all composition in thought or else something simple must be left over after its removal. But the first horn of the dilemma is supposed to be contradictory: in this case, "the composite would once again not consist of substances (because with substances composition is only a contingent relation, apart from which, as being persistent by themselves, they must subsist)" (ibid.). Therefore, concludes Kant, only the second case is left: the last elements of composite substances are simple.

The argument is strange. Even on a sympathetic reading, it is hard to see what constitutes its premise and what its consequence.[15] Once stripped of its—"redundant," as Hegel would say—proof form, it seems to revolve around two things: first, the factual question whether the initial object, presumably spatial matter, is to be considered substantial or not; and second, the definitional question whether the substantiality of a composite substance should be thought of as deriving from the substantiality of (some of) its parts or not. Read in this way, the alternatives should be clear and easy. If we assume that we started with a composite substance, and furthermore assume that, for its parts, being in relation is by definition something accidental, then it necessarily follows that the last elements of such a composite whole are simple.

But let us ask ourselves what prevents us to accept the other outcome of Kant's thought experiment, i.e., that after the removal of all composition nothing would be left over. The answer is, supposedly, that in that case *either* we could not assign substantiality to the initial object *or* we would have to allow for the possibility of composite substances that are essentially relational; that is, we would either have no true substance to start with or

15. It is often suggested that the argument consists in an arbitrary analysis of the notion of composite substance or that it is circular. For a closer assessment, see especially Engelhard (2005: 158ff). Falkenburg (1995: 17) speaks of "assuming mutually incompatible additional suppositions regarding composite substance 'in the world' that cannot be *fulfilled in one–and–the–same–model*".

the substance would dissolve after the removal of all relations. The problem is, though, that both options are in principle consistent and that both were actually defended, for instance by Leibniz. Thus he once claimed that matter was only a *phaenomenon bene fundatum*, which means that the argument concerning composite substances does not refer to it, and later, in his letters to Des Bosses, that matter could be made into something substantial by adding the so–called *vinculum substantiale*, in which case we could speak of a substance consisting of nothing but mere relations[16].

Even if Kant's proof thus again fails to convince the dogmatist, it does include a relevant point. The real question is how to conceive of substantiality, especially in relation to space and matter. It seems that, in Kant's view, true substantiality and spatiality exclude each other since substance is usually understood as something that is capable of existing by itself, without any relation to other substances, while space is essentially relational[17]. Or, better yet, Kant thought that this is the way dogmatists were bound to see the situation, given their concept of substance. He, on the other hand, was able to avoid the tension in question, he thought, since he disposed with the new concept of *substantia phenomenon*, which comprised many features of space, including its relational structure, yet was still supposed to function as something persistent and therefore substantial. In this sense, it may be said that, in the proof of the thesis, Kant actually operated with two different notions of composite substance,[18] and that the contradiction appeared only because he tried to impose his concept of material substance on dogmatists as he understood them. He thus only managed to demonstrate the following point: had Kant been a dogmatist, *he* would have contradicted himself.

Let us now turn to the antithesis. It reads:

> No composite thing in the world[19] consists of simple parts, and nowhere in it does there exist anything simple. (*KrV*, B463/A435)

In the proof, Kant assumes the contrary, namely the existence of simple parts, and continues:

16. De Risi and Graber noted a close similarity between Leibniz's late views on body and Kant's critical conception of material substance, so that in this respect Graber even speaks of Leibniz as "a transcendental philosopher" (2010: 362). De Risi suggests (2007: 314) that the Leibniz–Des Bosses correspondence was known to Kant; this, however, seems to be speculation only.

17. "If I remove all composition from it [i.e. space], then nothing, not even a point, might be left over" (*KrV*, B466/A438). See also R 5879 (1785–88; AA 18: 375) and R 5299 (1776–78; AA 18: 147).

18. In Kant's dilemma, "Either in substances it is impossible to remove all composition or else after its removal something simple must remain," both horns of the dilemma are completely acceptable: the first holds for the spatial substance and the second for the non–spatial one.

19. Significantly, in the antithesis Kant speaks of a "thing," not of a "composite substance" as before, thereby indicating that the context is spatial from the very beginning.

> Because every external relation between substances, hence every composition of them, is possible only in space, there must exist as many parts of space as there are parts of the composite occupying it. Now space does not consist of simple parts . . . (*Ibid*.)

The so–called proof (we will omit the second part relating to the spirits) consists in simply projecting the properties of space onto substances: because space does not consist of simple parts, no substance is simple; because space is infinitely divisible, the substance, too, is divisible to infinity. But the question is, precisely, whether the substance is really to be taken as essentially spatial. Once again, Kant simply *asserts* that which should actually be proved.

This time, however, he is somewhat aware that he is cheating. In the adjoining remark, he cites a serious objection, which is all too quickly (and invalidly) rejected by reference to the senses. That is, he mentions certain "monadists" who erroneously attempt to make the infinite divisibility of space compatible with the indivisibility of substances. "If one listens to them," Kant comments, "then besides mathematical points, which are simple, but are boundaries rather than parts of space, one would have to think of physical points too as being not only simple, but as also having, as parts of space, the privilege of filling it" (*KrV*, B467/A439). To whom is Kant referring? It would be wrong to think of Boscovich, for his physical points were located in a preexisting space, while according to Kant, these "monadists are subtle enough" to rather presuppose "these objects and the dynamical relation of substances in general as the condition of the possibility of space" (*KrV*, B469/A441).

In fact, this "absurd" monadist is none other than *Kant himself*! In 1756, he published the so–called *Physical Monadology*, in which he tried to "marry" metaphysics and mathematics as regards the infinite divisibility of spatial matter. And what did Kant do in 1756? He—perhaps under the influence of Knutzen—accepted the presupposition of metaphysics that matter was ultimately grounded in simple substances called monads, but avoided the usual complication by distinguishing between *esse in spatio* and *implere spatium*: "Each simple element of a body, that is to say, each monad, is not only in space; it also fills the space, though it does not, for that reason, forfeit its simplicity" (AA 1: 480). This is not to say that the monad is itself extended. In Leibniz's relational explanation, as we have seen, space arises only out of the *relations* existing among a certain *plurality* of monads. But, under such conditions, this non–extended substance can still have spatial effects[20], it can be present in a certain space and make it impenetrable. How?

20. Eueler developed a subtle theory of *virtual* spatiality of simple substances, which he called spirits. "Thus my soul does not exist in a certain place, but it acts in a certain place" (Euler 2003: 175). It is another question whether Kant was right in trying to ground space in unextended elements.

> The monad does not determine the little space of its presence by the plurality of its substantial parts, but by the sphere of activity, by means of which it hinders the things which are external to it and which are present to it on both sides from drawing any closer to each other. (AA 1: 480)

Every monad is endowed with an original active force that prevents other monads from entering into its *sphera activitatis*, thus limiting the sphere of activity of other, spatially neighboring monads. To counterbalance this repulsive force, Kant introduces the force of attraction, and even engages in speculation on the subject of their respective intensities in relation to distance from the central point. At the end, the consequence of it all is that this unextended and probably also non–spatial point can still have spatial effects that make its little space impenetrable for all the other monads and the space completely filled. And since the monad is not extended itself, only its effects are, one can in thought divide the space it fills without thereby affecting the simplicity of the monad.

Indeed, this infinite divisibility is not only imaginary or metaphysical, but also physical, at least in principle. We may be tempted to portray such a physical monad as a single solid ball, or as a bubble with a crusted envelope formed at the distance where the two internal forces cancel each other out. This picture is dangerously wrong, however, since the monad in question is by definition in relation to other monads with forces of their own. This means that its state is essentially dynamic, depending on the number and strength of the forces involved, and that consequently, for any given space that is presently filled by a certain monad, there is a greater external force that, when applied to it, would compress it to any desired dimension. The physical monad is fundamentally *elastic*. There is a limit to this elasticity, however: "It is obvious that by no conceivable force can an element be penetrated completely" (AA 1: 487). But that aside, according to this conception, material space is physically divisible to infinity, while the substances still retain their simplicity. Mathematics and metaphysics are married happily.

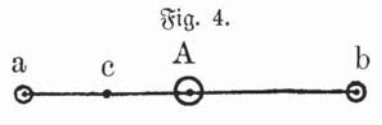
Having this in mind, there seems to be no reason why the dogmatist would need to renounce his position. At least in the *Critique of Pure Reason*, Kant provides no independent argument against physical monadology—which is strange, again, since he was obviously familiar with its strengths. It is only in the *Metaphysical Foundations of Natural Science*, published in 1786, that Kant finally developed a dedicated argument against the system in question. The initial situation here is very close to that in the

According to De Risi, Leibniz was indeed capable of showing that "space is actually constituted by points, even though it is not composed of them" (2007: 311); yet, at least in De Risi's view, Kant's early physical monadology should be "regarded as backward" compared to Leibniz.

second antinomy: in Explication 4 of the Dynamics, Kant similarly seeks to prove that matter, as the composite substance is called here, is divisible to infinity. This time, though, he is more thorough:

> The proof of the infinite divisibility of space has not yet come close to proving the infinite divisibility of matter, if it has not been previously shown that there is a material substance in every part of space that is, that parts movable in themselves are to be found there. (AA 4: 504)

Contrary to what he did in the proof of the second antithesis, Kant now openly admits that in order to refute the physical monadist, one first has to prove that every part of space is occupied by a substance. Here, too, Kant starts apagogically, drawing the following diagram:



Let us then assume that A is the site of a simple substance in space; and let us further assume that a and b are the points marking the opposite limits of the sphere of activity of substance A.[21] According to the infinite divisibility of space, Kant notes, it is possible to specify a point c somewhere on the line between central point A and border point a, i.e., within substance A's sphere of activity.

> If now A resists that which strives to penetrate into a, then c must also resist the two points A and a. For, if this were not so, they would approach one another without hindrance, and thus A and a would meet at the point c, that is, the space would be penetrated. Therefore, there must be something at c that resists the penetration of A and a . . . (AA 4: 504–505)

Hence it follows, Kant concludes, that in point c there must be a center of force repelling monad A, and that, therefore, in this point c a monad has to be found, too. Otherwise, monad A would not be able to resist external monads from penetrating into its sphere of activity, and contrary to the presupposition, its space would not be filled. That is to say, the space of substance A is filled only if at point c there is actually another monad present. And since the same reasoning applies to the monad at point c, or indeed at any point in the indefinitely divisible space, there must be a material substance in any given point of space. The physical monadist is refuted.

21. As Engelhard (2009: 331) notes, it is unclear what the circles around the points a, b, and A stand for. This may not be a trivial concern.

Is the argument sound? Commentators tend to disagree: many seem to accept it as valid; others claim that Kant has made certain tacit presuppositions in the process, for instance regarding the specific mode in which repulsive forces operate;[22] still others emphasize the complex structure of argumentation in the *Metaphysical Foundations*, where Kant conceives matter merely as the movable in space and insists on the relativity of motion.[23] So, how to proceed? If we simply follow the argument, we see that Kant first presupposes the existence of a simple, non–extended substance with point A as its center of force, which successfully repels other substances from penetrating into its sphere of activity. This substance thus dominates over its little space, which, to outside observers, appears to be perfectly filled. Inside this little space Kant now posits an arbitrary point, point c. Up to this moment, everything is fine. Now, however, Kant simply affirms that this point c *performs a certain activity on its own*, that it constitutes a real center and a subject of force itself. If anything, it is this affirmation that is contradictory since it was presupposed in the beginning that substance A fills its space and thereby *prevents* any other substance from entering it. This is *its space*, after all, that constitutes the sphere of its activity.

Kant makes his move on the ground that otherwise, that is, if there were no independent substance in point c, the points A and a would come together. But this is obviously false. We have a clear–cut dilemma: either substance A has a repulsive force or not. If it does have it, then, according to the hypothesis, it fills the space by itself. This space can always get bigger or smaller, true, but no matter how big the force applied against it may be, it will never squeeze it into an unextended point. If, on the other hand,

22. According to Kauark–Leite (2004: 142) and Schönfeld (2000: 171), for instance, Kant assumed that the repulsive force was a contact force acting on surfaces only. This is begging the question, it seems.

23. In his detailed discussion, Friedman emphasizes that since, according to the principle of relativity of space and motion, there is no absolute reference frame to determine motion, and since the exercise of force is considered to be instantaneous motion, "there is just as much reason to say c is resisting a and A . . . as there is to say that A is resisting a" (Friedman 2013: 151). It may be true, Friedman admits, that this reasoning no longer applies once the forces and causal relations are brought into the picture, for in this case (compare AA 4: 547) the privileged frame of reference is clearly determined by the center of mass. But, Friedman now observes, "although the central point A is initially taken to be at rest, the proper frame of reference for considering this particular exercise of repulsive force is actually centered on a point *between* A and a (and thus at the point c, for example)" (2013: 153). It is hard to see why this is so. The conclusion would hold, it seems, if point a constituted an independent source of repulsive force. However, since according to the initial assumption point a marks only a limit of substance A's sphere of activity, we have to assume that some *other* substance (call it E) is present which strives to penetrate into A's sphere at this point a (which, incidentally, could also be called e, since it also marks the limit of E's sphere of activity). In this case, it seems that the natural center of mass lies in the point a = e, and to put it anywhere else, an additional argument would be needed. Our reasoning suggests that Kant's diagram is (deliberately?) misleading in that it draws a circle around point a, presumably implying that it constitutes a force center of its own, that is, a substance.

substance A has no relevant force, then no number of such subjects will manage to fill the space in question, as Kant would have it. Anyhow, it was assumed that substance A does have the relevant force and that it does dominate over a certain space. Consequently, no other substance can be posited at point c within this space. Kant's proof is inconclusive, arbitrary, and in the end contradictory.

As regards the second antinomy, we finally have to conclude that for the dogmatist both thesis and antithesis once again remain standing. Leibniz would thus deny that infinitely divisible matter is anything more than a phenomenon, or else, relying on his late theory of *vinculum substantiale*, he would maintain that the composite substance called matter is essentially relational. As for our physical monadist, he would obviously continue to stick to his theory. In any event, all the doctrines mentioned above seem to be internally consistent, so that the question of their truth is finally empirical.

The situation is different for Kant, though. It is for him that, again, both proofs turn out to be conclusive—at least, that is, had he been a dogmatist. For in his constitution of an empirical world, Kant introduced a new concept of substance, one that was fundamentally defined in reference to the determination of time. As such, it remained underdetermined intrinsically. It is for instance extremely difficult to tell whether in Kant's world there is in the last analysis only *one* continuous substance, as there is only one time, or if there are *infinitely many* discrete substances, as there are so many parts matter can be divided into. It seems that, for various reasons, Kant was obliged to affirm both, thinking of substance as essentially relational *and* essentially self–standing at the same time. Paradoxically, however, it was not open to him to avoid the contradiction in question by referring to the standard distinction between the world as it is in itself and its appearance, so readily available to every dogmatist, because for him, the world of appearances was in a sense the only world there was.

3. The consequences

As we have seen, Kant's indirect proof has failed completely. Instead of showing how dogmatism is trapped in contradiction, the antinomy has exposed the contradictory nature of transcendental idealism itself. But, as we know, this is not critical for Kant, since he is supposed to provide an adequate cure for this self–inflicted wound.

According to Kant, the fundamental matrix of dialectical reasoning reads: "If the conditioned is given, then the whole series of all conditions for it is also given" (*KrV*, B525/A497). The dogmatist is justified in his conclusion,

Kant believes,[24] since for her the things are given independently of any subjective condition and are as such determined by the way they are in themselves. In the case of the world, this means that, with every element of it, the complete series of its conditions is given at the same time, be it finite or infinite. Here, therefore, one of the two options must be true: either the world has a first condition, which is unconditioned, or the unconditioned is the entire infinite whole taken together. For the transcendental idealist, however, the situation is different. Since for her the things of the world are mere appearances, they are never given in themselves, but first have to be constituted by the synthesis of cognition and are thus "given only *in this synthesis*" (B527/A499). The synthesis in question is *successive*, however. Although its general mechanism makes sure that with every step reached the synthesis could always continue, it can nonetheless never attain the infinite whole. Within this framework, the world as an infinite given whole is therefore a contradiction.[25] And according to the predicament of transcendental idealism, this can only mean that *there is no world*: the world simply does not exist!

The contradictory nature of the world explains why, for Kant, the proofs on both sides were actually valid. From contradiction, anything can be inferred. Kant's general solution to the problem of antinomy would thus be that the world is a forbidden object nothing can be predicated of.[26] Or alternatively—since the world is indeed spoken of—that it is *not a completely determined* object, in particular regarding its magnitude. "If it is said that the world is either infinite or finite (non–finite), then both propositions could be false," Kant remarks (*KrV*, B532/A504). Under the condition of indeterminacy, it is no longer allowed to reason that if the world is proven not to be finite, then it must be infinite, as was the case in the antinomic dispute.[27] Kant consequently concluded, first, that the world has no beginning in time, but for every past there is another past open to our exploration, without us ever being able to reach to infinity; and second, that matter can

24. See R 5553 (1778–79?; AA 18: 223): "The proposition that if the conditioned is given, the whole series of all conditions through which the conditioned is determined is also given is, if I abstract from the objects or take it merely intellectually, correct."

25. See R 4525 (1772–75?; AA 17: 582): "The absolute–whole in the appearance is a contradiction."

26. See R 5962 (1785–89; AA 18: 402): "The reason why the first two antinomies are both false is that I *had to* ground them both on a contradictory concept, namely, that of a whole in space and time that is also supposed to be an absolute whole."

27. The first antithesis, for instance, reads: "The world has no beginning and no bounds in space, but is infinite with regard to both time and space." According to Kant, the first part is actually valid and agrees with his own position. It is, however, invalid to conclude that the world is *therefore* infinite; to this effect, it should have first been proved that the world is actually determined in this respect. Here, and this is precisely the point of the so–called infinite judgment, the rule of the excluded third does not apply.

be decomposed into the infinite without itself consisting of infinitely many parts, since here, the parts are given only through the process of division, which can never reach infinity.

The proposed solution thus manages to pull Kant out of his self–inflicted contradiction. But since we have found that antinomy is inherent to transcendental idealism alone, it is not obvious that we are generally speaking in a better position than we started with. To conclude, we will therefore briefly consider where the source of the antinomy lies. In this respect, it might be helpful to recall that Kant discovered the phenomenon rather early in his philosophical development, but used to judge it quite differently at different stages. For instance, in 1769, the year that gave him "a great light," Kant interpreted this "evident self–contradiction" as the surest proof that the metaphysical concepts were merely subjective (AA 17: 357); in the Dissertation, in 1770, he explained the phenomenon as the "lack of accord between the *sensitive* faculty and the faculty of *understanding*," (AA 2: 389) where both logical regimes were considered to be equally *correct*, each in its proper sphere; and later, as he restricted the real use of understanding to the sensible world, he described the opposition in question as the conflict between "the partiality of sensibility and the totality of reason," declaring the thesis wrong and the antithesis right.[28] Only in the Critique, in 1781, did Kant return to symmetrical presentation, saying that both are wrong since they both make an impossible presupposition.[29]

In the year preceding the first Critique, Kant's treatment of antimony, it appears, revolved around the opposition between the abstract and the concrete, the intellectual and the sensitive, the conceptual and the intuitive, and even the total and the partial; he once deplored, for instance, that understanding often cannot expose its abstract ideas in the concrete and convert them into intuitions (see AA 2: 389). Considering this historical dynamism, we would thus propose that, in Kant, the contradiction between thesis and antithesis manifests the structural tension between the intellectual and the sensitive conditions of cognition. This was, in fact, long ago proposed by Maas (see Maas 1788: 441). If so, the antinomic conflict would be in principle generated simply by juxtaposing Kant's statements in the Transcendental Analytic and the Transcendental Aesthetic. In the Aesthetic, for instance, Kant claims that space is an infinite given whole, while in the Analytic he presents it as an axiom that everything in space is an extensive magnitude brought about by a successive aggregation of given parts. These two affirmations are clearly in opposition. Now, in Kant, since sensitivity

28. Compare, for instance, R 4760 (1775–77; AA 17: 711): "Thus there is actually no antinomy."

29. For a closer discussion, see Kreimendahl (1990), Guyer (1987), in particular pp. 387–404, and Kobe (1995), pp. 113–123 and 264–268.

and understanding are both necessary for cognition, the antinomy is unavoidable due to the very logic of cognition, not merely because of the trespassing of its boundaries: the contradiction arises here, in the middle of experience, not somewhere beyond it. Insofar as the logic of intuition and the logic of concept do not accord with each other, and insofar as cognition is defined precisely as a conjunction of intuition and concept, the antinomy is built into the very fabric of cognition.

However, such a rigid duality of aesthetics and analytics is still inadequate, as it misrepresents the structure of cognition, and especially as it does not pay appropriate attention to the adjustments made by Kant to the Critique, which, even if located outside the antimony chapter, significantly affect its implications. To put it in extremely short terms, in the second edition, Kant extended the reach of the spontaneity of understanding, cut its reference to any superior capacity, and made it into understanding that is "specific" to a finite rational being, such as a human. The contradiction now comes, as it were, from understanding alone. In his 1789 letter to Herz, Kant thus writes that "the antinomies of pure reason" could provide a good test stone . . . that one cannot assume human understanding to be of one kind with the divine" (AA 11: 54). We may say that the antinomy now has its source in the tension between the synthetic and analytic unities of consciousness, that is, in the incongruence between the universal and the particular inherent to every discursive understanding such as ours. This opposition may assume various guises, usually exploring the gap between the abstract and the concrete. For instance, it comes to the fore in Kant's observation: "I have a penny for every poor, but not for all the poors."[30] But since it arises out of the logical form of finite cognition and is built into the very notion of the object that joins together the unity of a thing with the plurality of its determinations, it may be said that for the transcendental idealist, every object is ultimately an antinomy.

A devoted Kantian would probably protest. For him, antinomies are strictly attached to the concept of the world and restricted to four, "no more, no less." However, when we examine the consequences of Kant's solution for the principle of complete determination, the proposition of universalized antinomy may regain its Kantian flavor. In traditional metaphysics, in Baumgarten for instance, this principle of *determinatio omnimoda* was held in high esteem, up to the point of being considered—under the heading of *complementum possibilitatis*—virtually synonymous with existence. In general, Kant seemed to agree with it. In the opening passages to his assessment of natural theology, he thus—approvingly, it appears—cites the principle in question, saying that "*everything existing is thoroughly determined*" (*KrV*,

30. R 4409 (1771?; AA 11: 535).

B601/A573). If Kant refused to consider it equivalent to existence, it was because he deemed it *too weak*, in the sense that not everything thoroughly determined is for that matter existent.[31]

Kant was well aware that this principle could never be exhibited *in concreto*, since "in order to cognize a thing completely one has to cognize everything possible and determine the thing through it" (ibid.). He did not seem to realize, however, at least not to the full extent, that by admitting this he subscribed to the conclusion that according to his theory—in exactly the same way as in the antinomies—the *existing objects are not completely determined*. Quite the opposite, for Kant, the things of this world are full of gaps, lacks, and cracks, and for every well–determined face they show us there is always a bottomless indeterminacy opening underneath them.[32]

By examining the consequences of Kant's presentation of antinomy, we have finally ended in a strange world. In a world where due to the logical form of finite understanding everything harbors a contradiction, where things are penetrated by gaps in the very fabric of their being, with an infinite abyss of indeterminacy gaping beneath them, and where they are therefore supported in their being only by a self–sustaining web of reasons. In short, we arrive at a suspended gappy ontology, quite similar to the one recently attributed to Hegel (see Bowman 2013, Kreines 2015).

Kant wanted to present the antinomy in the instrumental fashion as an indirect proof of transcendental idealism. By examining the two mathematical antinomic conflicts we tried to show that his attempt was a multi–layer failure. What is remarkable, however, is the inner structure of this failure, its conditions of possibility, and the consequences it brings along. We have seen, for instance, that Kant must have known perfectly well that the dogmatist would reject his arguments. We have also seen that it was precisely the transcendental idealist who found herself under the threat of contradiction, as her conception of the object of experience included conflicting requirements. It was her who desperately needed a solution to exit from this self–inflicted wound. But if, consequently, the antinomy has to be read as a symptom of transcendental idealism as such, then its solution should also be understood in this general sense, that is, ontologically (especially since for the transcendental idealist there was ultimately but one world).

31. "Everything that exists is thoroughly determined; but it is not this thoroughgoing determination that constitutes the concept of existence, rather that a thing is posited absolutely and not merely in relation to its concept." (R 5710; 1780s?; AA 18: 332) Compare also R 6256 (1783–84; AA 18: 533) and R 6322 (1792–94; AA 18: 638). For a closer examination, see Engelhardt (2005), pp. 321–334, and Klimmek (2005), pp. 165–180.

32. This is not to say that we could somehow observe their gaps, for by bringing the cognitive synthesis further, the gap of indeterminacy gets *eo ipso* filled. However, in exactly the same way as in the antinomy, we know *a priori* that existing things are not completely determined.

This is a Hegelian point, obviously. However, the difference is, we hope, that it was made here following Kant's arguments and according to his criteria.

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